CHAPTER

# 1

# SETS

*Animation 1.1: Sets-math*
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## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Express a set in:
  - descriptive form,
  - set builder form,
  - tabular form.
- Define union, intersection and difference of two sets
- Find:
  - union of two or more sets,
  - intersection of two or more sets,
  - difference of two sets
- Define and identify disjoint and overlapping sets
- Define a universal set and compliment of a set
- Verify different properties involving union of sets, intersection of sets, difference of sets and compliment of a set, e.g $A \cap A' = \phi$.
- Represent sets through Venn diagram.
- Perform operation of union, intersection, difference and complement on two sets A and B, when:
  - A is subset of B,
  - B is subset of A,
  - A and B are disjoint sets,
  - A and B are overlapping sets, through Venn diagram.

## 1.1 Introduction

In our daily life, we use the word set only for some particular collections such as water set, tea set, dinner set, sofa set, a set of books, a set of colours and so on.

But in mathematics, the word set has broader meanings than those in our daily life because it provides us a way to integrate the different branches of mathematics.





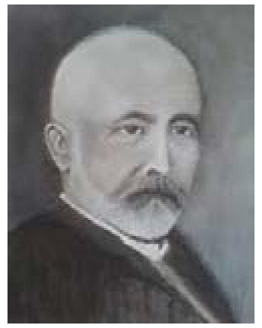
It also helps to solve many mathematical problems of both simple and complex nature. In short, it plays a pivotal role in the advanced study of the mathematics in the modern age. Look at the following examples of a set.

A = The set of counting numbers.
B = The set of Pakistani Provinces.
C = The set of geometrical instruments

**Recall**

**A set cannot consist of elements like moral values, concepts, evils or virtues etc.**

" A set is a collection of well defined objects/numbers. The objects/numbers in any set are called its members or elements"



"Set theory" is a branch of mathematics that studies sets. It is the creation of George Cantor who was born in Russia on March 03, 1845. In 1873, he published an article which makes the birth of set theory. George Cantor died in Germany on January 06, 1918

### 1.1.1 Expressing a Set

There are three ways to express a set.

1. Descriptive form 2. Tabular form 3. Set builder form

- **Descriptive form**

If a set is described with the help of a statement, it is called as descriptive form of a set.

**For Example:**

N = set of natural numbers
Z = set of integers
P = set of prime numbers
W = set of whole numbers
S = set of solar months start with letter "J"

**Do you Know**

**The sets of natural numbers, whole numbers, integers, even numbers and odd numbers are denoted by the English letters N, W, Z, E and O respectively.**

- **Tabular Form**

If we list all elements of a set within the braces { } and separate each element by using a comma "**,**" it is called the tabular or roster form.

**For Example:**

A = {*a, e, i, o, u*}  C = {3, 6, 9, ... ,99}
M = {*football, hockey , cricket*}  N = {1, 2, 3, 4, ...}
W = {0, 1, 2, 3, ...}  X = {*a, b, c, ..., z*}

- **Set Builder Form**

If a set is described by using a common property of all its elements, it is called as set builder form. A set can also be expressed in set builder form. For example , "E is a set of even number" in the descriptive form, where E = {0, ±2, ±4, ±6,.....} is the tabular form of the same set. This set in set builder form can be written as;

**E = {$x$ | $x$ is an even number}**

and we can read it as, E is a set of elements $x$, such that $x$ is an even number.

A = {$x$ | $x$ is a solar month of a year}
B = {$x$ | $x \in$ N $\wedge$ 1 < $x$ < 5}
C = {$x$ | $x \in$ W $\wedge$ x $\leq$ 4}

**Some Important Symbols**

| | |
|---|---|
| \| such that | $\in$ belongs to |
| $\wedge$ and | $\vee$ or |
| $\geq$ greater than or equal to | |
| $\leq$ less than or equal to | |





## EXERCISE 1.1

**1. Write the following sets in descriptive form.**

(i) A = {*a, e, i, o, u*}  (ii) B = {3, 6, 9, 12, ...}
(iii) C = {*s, p, r , i, n, g*}  (iv) D = {*a, b, c, ... ,z*}
(v) E = {6, 7, 8, 9, 10}  (vi) F = {0, ±1, ±2}
(vii) G = {$x$ | $x \in$ N $\wedge$ $x$ < 3}  (viii) H = {$x$ | $x \in$ N $\wedge$ $x$ > 99}

**2. Write the following sets in tabular form.**

(i) A = Letters of the word "hockey"
(ii) B = Two colours in the rainbow
(iii) C = Numbers less than 18 divisible by 3
(iv) D = Multiples of 5 less than 30
(v) E = {$x$ I $x \in$ W $\wedge$ x > 5}
(vi) F = {$x$ I $x \in$ Z $\wedge$ – 7 < $x$ < –1}

**3. Write the following sets in the set builder form.**

(i) A = {1,2,3,4,5}
(ii) B = {2,3,5,7}
(iii) N = set of natural numbers
(iv) W = set of whole number
(v) Z = set of all integers
(vi) L = {5, 10, 15,20,...}
(vii) E = set of even numbers between 1 and 10
(viii) O = set of odd numbers greater than 15
(ix) C = set of planets in the solar system
(x) S = set of colours in the rainbow

# 1.2 Operations on Sets

## 1.2.1 Union, Intersection and Difference of Two Sets

- **Union of Two Sets**

The union of two sets A and B is a set consisting of all the elements which are in set A or set B or in both. The union of two sets is denoted by $A \cup B$ and read as "A union B"

**Example 1:** If $A = \{a, e, i, o\}$ and $B = \{a, b, c\}$, then find $A \cup B$

**Solution:**

$A = \{a, e, i, o\}, B = \{a, b, c\}$

$A \cup B = \{a, e, i, o\} \cup \{a, b, c\}$

$= \{a, e, i, o, a, b, c\}$

**Example 2:** If $M = \{1, 2, 3, 4, 5\}$ and $N = \{1, 3, 5, 7\}$, then find $M \cup N$

**Solution:**

$M = \{1, 2, 3, 4, 5\}, N = \{1, 3, 5, 7\}$

$M \cup N = \{1, 2, 3, 4, 5\} \cup \{1, 3, 5, 7\}$

$= \{1, 2, 3, 4, 5, 7\}$

- **Intersection of Two Sets**

The intersection of two sets A and B is a set consisting of all the common elements of the sets A and B. The intersection of two sets A and B is denoted by $A \cap B$ and read as "A intersection B"

**Example 3:** If $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d, e\}$, then find $A \cap B$

**Solution:**

$A = \{a, e. i, o, u\}, B = \{a, b, c, d, e\}$

$A \cap B = \{a, e, i, o, u\} \cap \{a, b, c, d, e\}$

$= \{a, e\}$

*Animation 1.2: Intersection of two sets*
*Source & Credit: elearn.punjab*





**Example 4:** If $X = \{1, 2, 3, 4\}$ and $Y = \{2, 4, 6, 8\}$, then find $X \cap Y$

**Solution:**

$X = \{1, 2, 3, 4\}, Y = \{2, 4, 6, 8\}$

$X \cap Y = \{1, 2, 3, 4\} \cap \{2, 4, 6, 8\}$

$= \{2, 4\}$

- **Difference of Two Sets**

Consider A and B are two any sets, then A difference B is the set of all those elements of set A which are not the elements of set B. It is written as $A - B$ or $A \setminus B$. Similarly, B difference A is the set of all those elements of set B which are not the elements of set A. It is written as $B - A$ or $B \setminus A$.

**Example 5:** If $A = \{1, 3, 6\}$ and $B = \{1, 2, 3, 4, 5\}$, then find:
(i) $A - B$ (ii) $B - A$

**Solution:**

$A = \{1, 3, 6\}, B = \{1, 2, 3, 4, 5\}$

(i) $A - B = \{1, 3, 6\} - \{1, 2, 3, 4, 5\}$

$= \{6\}$

(ii) $B - A = \{1, 2, 3, 4, 5\} - \{1, 3, 6\}$

$= \{2, 4, 5\}$

## 1.2.2 Union and Intersection of Two or More Sets

We have learnt the method for finding the union and intersection of two sets. Now we try to find the union and intersection of three sets.

- **Union of three sets**

Following are the steps to find the union of three sets

**Step 1:** Find the union of any two sets.
**Step 2**: Find the union of remaining 3rd set and the set that we get as the result of the first step
For three sets A, B and C their union can be taken in any of the following ways.

**(i)** $A \cup (B \cup C)$ **(ii)** $(A \cup B) \cup C$

It will be easier for us to understand the above method with examples. Look at the given examples.

**Example 6:** Find A ∪ (B ∪ C) where A = {1, 2, 3, 4}, B = {3, 4, 5, 6, 7, 8} and C = {6, 7, 8, 9, 10}.

**Solution:**

A ∪ (B ∪ C) = {1, 2, 3, 4} ∪ {3, 4, 5, 6, 7, 8} ∪ {6, 7, 8, 9, 10}
= {1, 2, 3, 4} ∪ {3, 4, 5, 6, 7, 8, 9, 10}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

**Example 7:** If A = {1, 3, 7}, B = {3, 4, 5} and C = {1, 2, 3, 6}

**Solution:**

(A ∪ B) ∪ C = {1, 3, 7} ∪ {3, 4, 5}) ∪ {1, 2, 3, 6}
= {1, 3, 4, 5, 7} ∪ {1, 2, 3, 6}
= {1, 2, 3, 4, 5, 6, 7}

- **Intersection of Three Sets**

For finding the intersection of three sets, first we find the intersection of any two sets of them and then the intersection of the 3rd set with the resultant set already found.

**(i)** A ∩ (B ∩ C) **(ii)** (A ∩ B) ∩ C

**Example 8:** Find A ∩ (B ∩ C) where A = {*a, b, c, d*}, B = {*c, d, e*} and C = {*c, e, f, g*}

**Solution:**

A ∩ (B ∩ C) = {*a, b, c, d*} ∩ ({*c, d, e*} ∩ {*c, e, f, g*})
= {*a, b, c, d*} ∩ {*c, e*}
= {*c*}

**Example 9:** If A = {1, 2, 3, 4}, B= {2, 3, 4, 5} and C = {1, 2}, then find (A ∩ B) ∩ C

**Solution:**

(A ∩ B) ∩ C = ({1, 2, 3, 4} ∩ {2, 3, 4, 5}) ∩ {1, 2}





= {2, 3, 4} ∩ {1, 2} = {2}

## EXERCISE 1.2

**1.** Find the union of the following sets.

(i) A = {1,3,5}, B = {1,2,3,4}
(ii) S = {*a, b, c*}, T = {*c, d, e*}
(iii) X = {2,4,6,8,10}, Y = {1,5,10}
(iv) C = {*i, o, u*}, D = {*a, e, o*}, E = {*i, e, u*}
(v) L = {3, 6, 9, 12}, M = {6, 12, 18, 24,}, N = { 4, 8, 12, 16}

**2.** Find the intersection of the following sets.

(i) P = {0, 1, 2, 3}, Q = {–3, –2, –1, 0}
(ii) M = {1, 2, ... , 10}, N = {1, 3, 5, 7, 9}
(iii) A = {3, 6, 9, 12, 15}, B = {5, 10, 15, 20}
(iv) U = {–1, –2 , –3}, V = {1, 2, 3}, W = {0, ±1, ±2}
(v) X = {*a, l, m*}, Y = {*i, s, l, a, m*}, Z = {*l, i, o, n*}

**3.** If N = set of Natural numbers and W = set of Whole numbers, then find N U W and N ∩ W

**4.** If P = set of Prime numbers and C = set of Composite numbers, then find P U C and P ∩ C

**5.** If A = {a, c, d, f}, B = {b, c, f, g} and C = {c, f, g, h}, then find
(i) A U (B U C) (ii) A ∩ (B ∩ C)

**6.** If X = {1, 2, 3, ....., 10}, Y = {2, 4, 6, 8, 12} and Z = {2, 3, 5, 7, 11}, then find:
(i) X U (Y U Z) (ii) X ∩ (Y ∩ Z)

**7.** If R = {0, 1, 2, 3}, S = [0, 2, 4) and T = {1, 2, 3, 4}, then find:
(i) R \ S (ii) T \ S (iii) R \ T (iv) S \ R

### 1.2.3 Disjoint and Overlapping Sets

- **Disjoint Sets**

Two sets A and B are said to be disjoint sets, if there is no common element between them. In other words there intersection is an empty set, i.e. A ∩ B = ϕ. For example, A = {1, 2, 3}, B = {4, 5, 6} are disjoint sets because there is no common element in set A and B.

- **Overlapping Sets**

Two sets A and B are called overlapping sets, if there is at least one element common between them but none of them is a subset of the other. In other words, their intersection is non-empty set. For example, A = {0, 5, 10} and B= {1, 3, 5, 7} are overlapping sets because 5 is a common element in sets A and B and non is subset of the other.

## 1.2.4 Universal Set and Complement of a Set

- **Universal Set**

A set which contains all the possible elements of the sets under consideration is called the universal set. For example, the universal set of the counting numbers means a set that contains all possible numbers that we can use for counting. To represent such a set we use the symbol U and read it as "Universal set" i.e.

The universal set of counting numbers: U = {1, 2, 3, 4, ...}

- **Complement of a Set**

Consider a set B whose universal set is U, then the difference set U \ B or U - B is called the complement of a set B, which is denoted by B′ or $B^c$ and read as "B complement". So, we can define the complement of a set B as: "B complement is a set which contains all those elements of universal set which are not the elements of set B, i.e. B′ = U \ B.

**Example 1:** If U = {1, 2, 3, ..., 10} and B = {1, 3, 7, 9}, then find B′.

**Solution:**

U = {1, 2, 3, ...,10}, B = {1, 3, 7, 9}
B′ = U – B
= {1, 2, 3, ... , 10} – {1, 3, 7, 9}
= {2, 4, 5, 6, 8, 10}





## EXERCISE 1.3

**1.** Look at each pair of sets to separate the disjoint and overlapping sets.

| | | | |
|---|---|---|---|
| (i) A = | {*a, b, c, d, e*}, | B = | {*d, e, f, g, h*} |
| (ii) L = | {2, 4, 6, 8, 10}, | M = | {3, 6, 9, 12} |
| (iii) P = | Set of Prime numbers, | C = | Set of Composite numbers |
| (iv) E = | Set of Even numbers, | O = | Set of Odd numbers |

**2.** If U = {1, 2, 3, ...., 10}, A = {1, 2, 3, 4, 5}, B = {1, 3, 5, 7, 9}, C = {2, 4, 6, 8, 10} and D = 3, 4, 5, 6, 7}, then find:
(i) A′ (ii) B′ (iii) C′ (iv) D′

**3.** If U = {*a, b, c,...., i* }, X = {*a, c, e, g, i*}, Y = {*a, e, i*}, and Z = {*a, g, h*}, then find:
(i) X′ (ii) Y′ (iii) Z′ (iv) U′

**4.** If U= {1, 2, 3, ..., 20}, A= {1, 3, 5, ... ,19} and B = {2, 4, 6, ... ,20}, then prove that:
(i) B′ = A (ii) A′ = B (iii) A \ B = A (iv) B \ A = B

**5.** If U = set of integers and W = set of whole numbers, then find the complement of set W.

**6.** If U = set of natural numbers and P = set of prime numbers, then find the complement of set P.

## 1.2.5 Properties involving Operations on Sets

We have learnt the four operations of sets, i.e. union, intersection, difference and complement. Now we discuss their properties.

- **Properties involving Union of Sets**

- **Commutative property**

If A, B are any two sets, then "**A ∪ B = B ∪ A**" is called the commutative property of union of two sets.

**Example 1:** If A= {1, 2, 3} and B = {2, 4, 6}, then verify that:
A ∪ B = B ∪ A.

**Solution:**

A U B = {1, 2, 3} ∪ {2, 4, 6}
= {1, 2, 3, 4, 6}
B U A = {2, 4, 6} ∪ {1, 2, 3}
= {1, 2, 3, 4, 6}

From the above, it is verified that:

A ∪ B = B ∪ A

- **Associative Property**

If A, B and C are any three sets, then "A ∪ (B ∪ C) = (A ∪ B) ∪ C" is called the associative property of union of three sets.

**Example 2:** If A = {1, 2, 3, 4, 5}, B = {1, 3, 5, 7} and C = {2, 4, 6, 8}, then verify that: A ∪ (B ∪ C) = (A ∪ B) ∪ C

**Solution:**

L.H.S = A ∪ (B ∪ C)
= {1, 2, 3, 4, 5} ∪ ({1, 3, 5, 7} ∪ {2, 4, 6, 8})
= {1, 2, 3, 4, 5} ∪ {l, 2, 3, 4, 5, 6, 7, 8}
= {1, 2, 3, 4, 5, 6, 7, 8}

R.H.S = (A ∪ B) ∪ C
= ({1, 2, 3, 4, 5} ∪ {1, 3, 5, 7}) ∪ {2, 4, 6, 8}
= {1, 2, 3, 4, 5, 7} ∪ {2, 4, 6, 8}
= {1, 2, 3, 4, 5, 6, 7, 8}

We see that L.H.S = R.H.S

- **Identity Property with respect to Union**

In sets, the empty set ϕ acts as identity for union, i.e. A ∪ ϕ = A

**Example 3:** If A = {*a, e, i, o, u*}, then verify that A ∪ ϕ = A.

**Solution:**

A ∪ ϕ = A
L.H.S = A ∪ ϕ





= {*a, e, i, o, u*} U { }
= {*a, e, i, o, u*} = A = R.H.S

Hence proved: L.H.S = R.H.S

- **Properties involving Intersection of Sets**

- **Commutative Property**

If A, B are any two sets, then

A ∩ B = B ∩ A

is called the commutative property of intersection of two sets.

**Example 4:** If a = {a, b, c, d} and B = {a, c, e, g}, then verify that A ∩ B = B ∩ A.

**Solution:**

A ∩ B = {a, b, c, d} ∩ {a, c, e, g}
= {a, c}
B ∩ A = {a, c, e, g} ∩ {a, b, c, d}
= {a, c}

From the above it is verified that A ∩ B = B ∩ A.

**Example 5:** If A= {1, 2, 3} and B = {4, 5, 6}, then verify that A ∩ B = B ∩ A.

**Solution:**

A ∩ B = {1, 2, 3} ∩ {4, 5, 6}
= { }
B ∩ A = {4, 5, 6} ∩ {1, 2, 3}
= { }

From the above it is verified that A ∩ B = B ∩ A.

- **Associative Property**

If A, B and C are any three sets, then A ∩ (B ∩ C) = (A ∩ B) ∩C is called the associative property of intersection of three sets.

**Example 6:** If A = {1, 2, 5, 8}, B = {2, 4, 6} and C = {2, 4, 5, 7}, then verify that: A ∩ (B ∩ C) = (A ∩B) ∩ C

**Solution:**

A = {1, 2, 5, 8}, B = {2, 4, 6}, C = {2, 4, 5, 7}

L.H.S = A ∩ (B ∩ C)

= {1, 2, 5, 8} ∩ ({2, 4, 6} ∩ {2, 4, 5, 7})

= {l, 2, 5, 8} ∩ {2, 4} = {2}

R.H.S = (A ∩ B) ∩ C

= ({l, 2, 5, 8} ∩ {2, 4, 6}) ∩ {2, 4, 5, 7}

= {2} ∩ {2, 4, 5, 7} = {2}

It is verified that L.H.S = R.H.S

- **Identity Property with respect to Intersection**

In sets, the universal set U acts as identity for intersection, i.e. A ∩ U = A.

**Example 7:** If U = {*a, b, c, ..., z*} and A= {*a, e, i, o, u*}, then verify that A ∩ U = A.

**Solution:**

U = {a, b, c, ...,z}, A= {a, e, i, o, u}

L.H.S = A ∩ U

= {a, e, i, o, u} ∩ {a, b, c, ..., z)

= {a, e, i, o, u}= A= R.H.S

Hence verified that L.H.S = R.H.S

- **Properties involving Difference of Sets**

If A and B are two unequal sets, then A – B ≠ B – A, For example if A = {0, 1, 2) and B = {1, 2, 3}, then

A – B = {0, 1, 2} – {1, 2, 3}

= {0}

B – A = {1, 2, 3} – {0, 1, 2}

= {3}

We can see that A – B ≠ B – A

- **Properties involving Complement of a Set**

Properties involving the sets and their complements are given below

A′ ∪ A = U     A ∩ A′ = $\phi$     U′ = $\phi$     $\phi$′ = U





**Example 8:** If U = {1, 2, 3, ... ,10} and A= {1, 3, 5, 7, 9}, then prove that:

**(i)** U′ = $\phi$   **(ii)** A ∪ A′ = U   **(iii)** A ∩ A′ = $\phi$   **(iv)** $\phi$′ = U

**Solution:**

U = {1, 2, 3, ..., 10}, A= {1, 3, 5, 7, 9}

**(i): U′ =** $\phi$

L.H.S = U′

We Know that U′ = U – U

= {1, 2, 3, ..., 10} - {1, 2, 3, ..., 10}

= { } = R.H.S

Hence verified that L.H.S = R.H.S

**(ii): A U A′ = U**

We know that A′ = U – A

= {1, 2, 3, ..., 10} - {1, 3, 5, 7, 9}

= {2, 4, 6, 8, 10}

Now we find,

A U A′ = {1, 3, 5, 7, 9} U {2, 4, 6, 8, 10}

= {1, 2, 3, ..., 10} = R.H.S

Hence verified that L.H.S = R.H.S

**(iii) A ∩ A′ =** $\phi$

L.H.S = A ∩ A′

= {1, 3, 5, 7, 9} ∩ {2, 4, 6, 8, 10}

= { } = $\phi$ = R.H.S

Hence verified that L.H.S = R.H.S

**(iv)** $\phi$**′ = U**

We know that

$\phi$′ = U – $\phi$

= {1, 2, 3, ... ,10} – { }

= {1, 2, 3, ... ,10} = U = R.H.S

Hence verified that L.H.S = R.H.S

## EXERCISE 1.4

**1.** If A= {a, e, i, o, u}, B = {a, b, c} and C = {a, c, e, g}, then verify that:

(i) $A \cap B = B \cap A$ (ii) $A \cup B = B \cup A$ (iii) $B \cup C = C \cup B$
(iv) $B \cap C = C \cap B$ (v) $A \cap C = C \cap A$ (vi) $A \cup C = C \cup A$

**2.** If $X = \{1, 3, 7\}$, $Y = \{2, 3, 5\}$ and $Z = \{1, 4, 8\}$, then verify that:
(i) $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (ii) $X \cup (Y \cup Z) = (X \cup Y) \cup Z$

**3.** If $S = \{-2, -1, 0, 1\}$, $T = \{-4, -1, 1, 3\}$ and $U = \{0, \pm1, \pm2\}$, then verify that:
(i) $S \cap (T \cap U) = (S \cap T) \cap U$ (ii) $S u (T \cup U) = (S \cup T) \cup U$

**4.** If $O = \{1, 3, 5, 7.....\}$, $E = \{2, 4, 6, 8......\}$ and $N = \{1, 2, 3, 4....\}$, then verify that:
(i) $O \cap (E \cap N) = (O \cap E) \cap N$ (ii) $O \cup (E \cup N) = (O \cup E) \cup N$

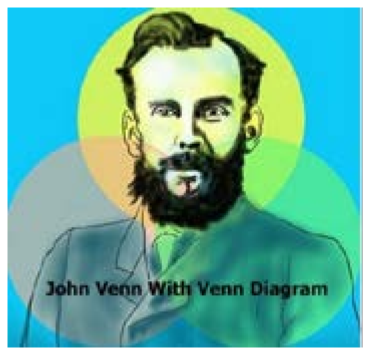
**5.** If $U = \{a, b, c, ....,z\}$, $S = \{a, e, i, o, u\}$ and $T = \{x, y, z\}$, then verify that:
(i) $S \cup \phi = S$ (ii) $T \cap U = T$ (iii) $S \cap S' = \phi$ (iv) $T \cup T' = U$

**6.** If $A = \{1, 7, 9, 11\}$, $B = \{1, 5, 9, 13\}$, and $C = \{2, 6, 9, 11\}$, then verify that:
(i) $A - B \neq B - A$ (ii) $A - C \neq C - A$

**7.** If $U = \{0, 1, 2,.....,15\}$, $L = \{5, 7, 9,.....,15\}$, and $M = \{6, 8, 10, 12, 14\}$, then verify the identity properties with respect to union and intersection of sets.

## 1.3 Venn Diagram

A Venn diagram is simple closed figures to show sets and the relationships between different sets.




Venn diagram were introduced by a British logician and philosopher "John Venn" (1834 - 1923). John himself did not use the term "Venn diagram" Another logician "Lewis" used it first time in book "A survey of symbolic logic"

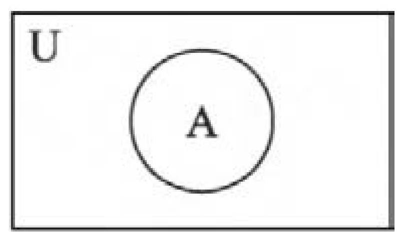




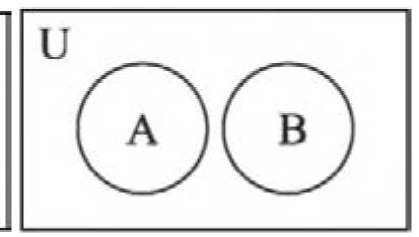
*Animation 1.3: Venn diagram*
*Source & Credit: elearn.punjab*

### 1.3.1 Representing Sets through Venn diagrams

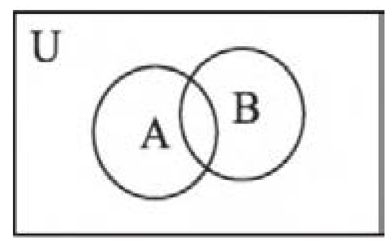
In Venn diagram, a universal set is represented by a rectangle and the other sets are represented by simple closed figures inside the rectangle. These closed figures show an overlapping region to describe the relationship between them. Following figures are the Venn diagrams for any set A of universal set U, disjoint sets A and B and overlapping sets A and B respectively.



*Set A*



*Disjoint Sets*



*Overlapping Sets*

In the Venn diagram, the shaded region is used to represent the result of operation.

### 1.3.2 Performing Operation on Sets through Venn Diagram

- **Union of Sets**

Now we represent the union of sets through Venn diagram when:

- **A is subset of B**

When all the elements of set A are also the elements of set B, then we can represent A U B by (figure i). Here shaded portion represents A U B.

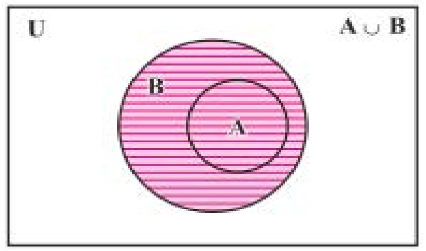



*Figure (i)*

- **B is subset of A**

When all the elements of set B are also the elements of set A, then we can represent A U B by (figure ii). Here shaded portion represents A U B.

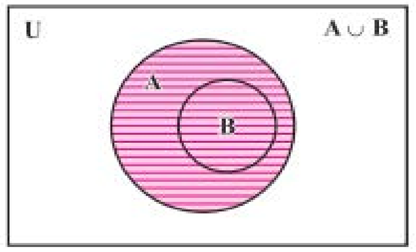



*Figure (ii)*

- **A and B are overlapping Sets**

When only a few elements of two sets A and B are common, then they are called overlapping sets. A U B is represented by (figure iii). Here shaded portion represents A U B.





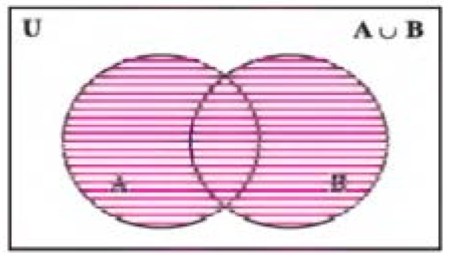



*Figure (iii)*

- **A and B are disjoint Sets**

When no element of two sets A and B is common, then we can represent A U B by (figure iv). Here shaded portion represents A U B.

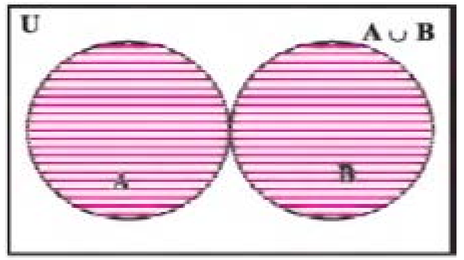



*Figure (iv)*

- **Intersection of Sets**

Now we clear the concept of intersection of two sets by using Venn diagram. In the given figures the shaded portion represents the intersection of two sets when:

- **A is subset of B**

When all the elements of set A are also the elements of set B, then we can represent A ∩ B by (figure v). Here shaded portion represents A ∩ B.

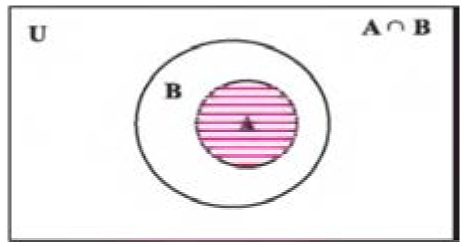



*Figure (v)*

- **B is subset of A**

When all the elements of set B are also the elements of set A, then we can represent A ∩ B by (figure vi). Here shaded portion represents A ∩ B.

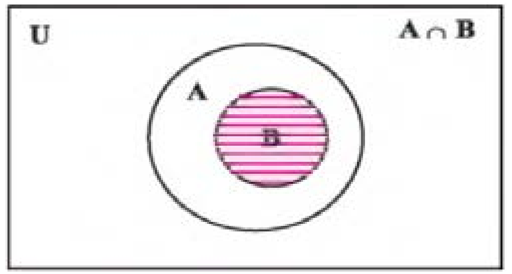



*Figure (vi)*

- **A and B are overlapping Sets**

When some elements are common, then we can represent A ∩ B by the fig (vii).

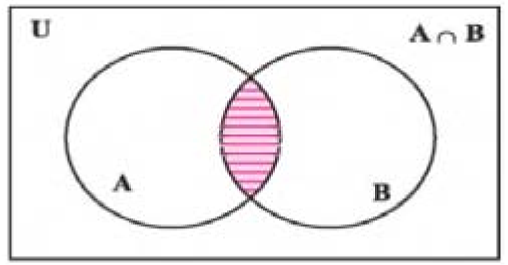



*Figure (vii)*

- **A and B are disjoint Sets**

When no element is common, then we can represent A ∩ B by fig (viii). So A ∩ B is an empty set.

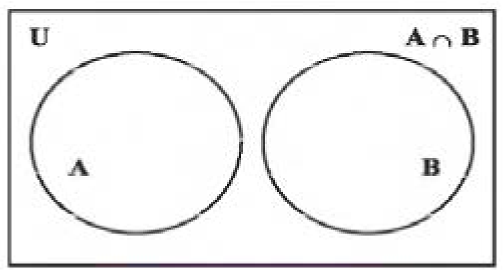



*Figure (viii)*

- **Difference of Two Sets A and B**

It is represented by shaded portion when:





- **A is subset of B**

When all the elements of set A are also the elements of set B, then we can represent A – B by fig (ix).

There is no shaded portion. So, A – B = { }

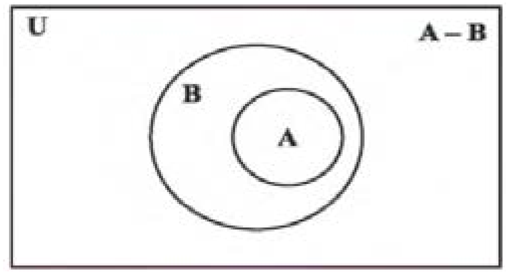



*Figure (ix)*

- **B is subset of A**

When all the elements of set B are also the elements of set A, then we can represent A - B by (figure x). Here shaded portion represents A - B.

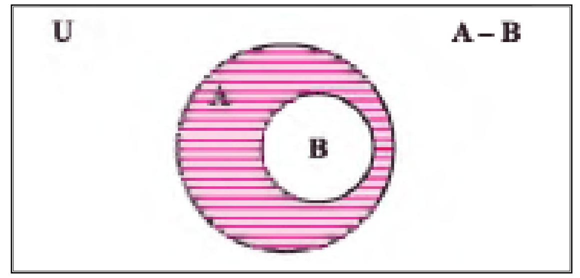



*Figure (x)*

- **A and B are overlapping Sets**

When some elements are common, then we can represent A – B by the fig (xi).

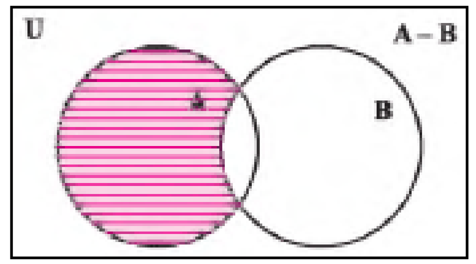



*Figure (xi)*

- **A and B are disjoint Sets**

When no element of two sets A and B is common, then we can represent A – B by (figure xii). Here shaded portion represents A – B.

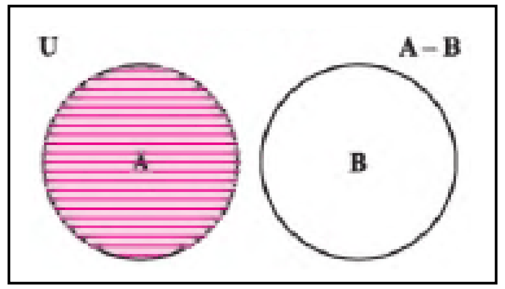
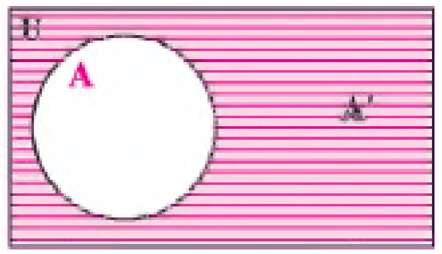



*Figure (xii)*

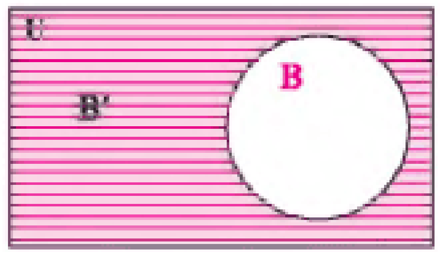
## 1.3.3 Complement of a Set

For complement of a set A




*U – A = A'*

For complement of a set B

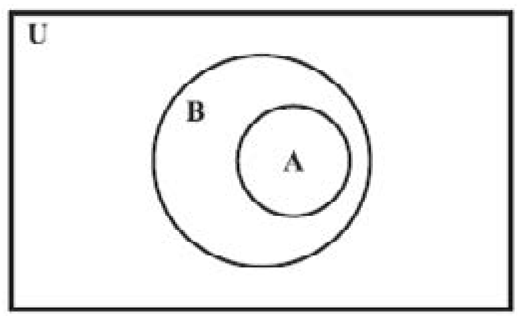



*U – B = B'*

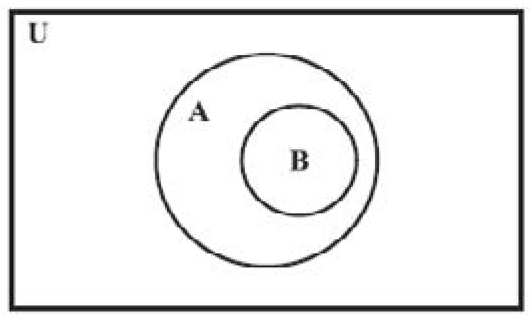
## EXERCISE 1.5

**1.** Shade the diagrams according to the given operations.

(i) A ∩ B (A is subset of B)



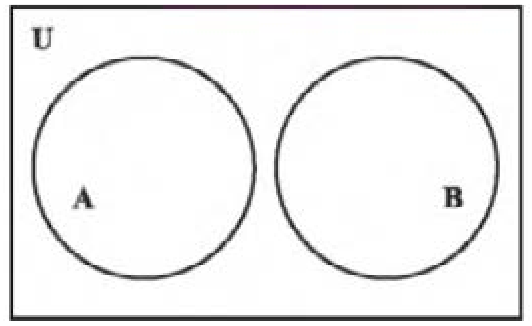

(ii) A ∪ B (B is subset of A)



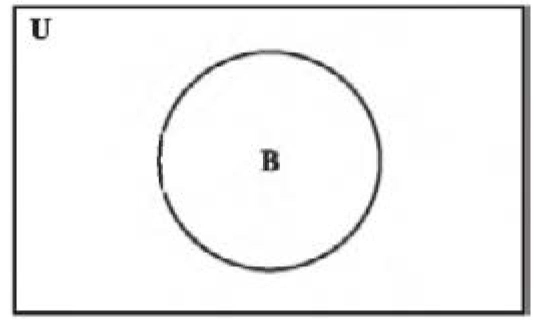




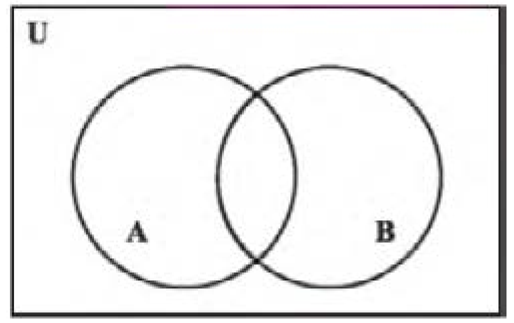

(iii) A – B (For disjoint sets)



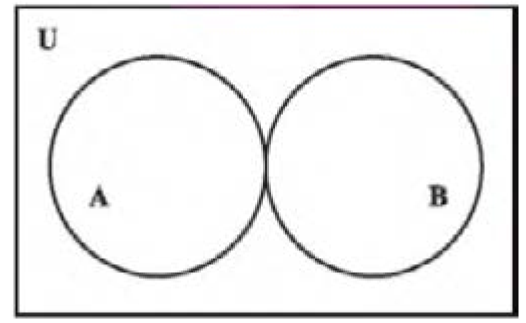

(iv) B'




(v) A ∩ B (Overlapping sets)




(vi) A ∪ B ( For disjoint sets)




**2.** If, U= {1, 2, 3, 10}, A={1,4, 8, 9, 10} and B = {2, 3, 4, 7, 10}, then show that:

(i) A – B ≠ B – A (ii) A ∩ B = B ∩ A
(iii) A U B = B U A (iv) A' ≠ B'
through Venn diagram.

## Review Exercise 1

**1.** Answer the following questions.

(i) Name three forms for describing a set.
(ii) Define the descriptive form of set.
(iii) What does the symbol " | " mean ?
(iv) Write the name of the set consisting of all the elements of given sets under consideration.
(v) What is meant by disjoint sets?

**2.** Fill in the blanks.

(i) The symbol " ^ " means _______.

(ii) The set consisting of only common elements of two sets is called the _______ of two sets.

(iii) A set which contains all the possible elements of the sets under consideration is called the _______ set.

(iv) Two sets are called _______ if there is at least one element common between them and non of the sets is subset of the other.

(v) In sets, the universal set acts as _______ for intersection.

**3.** Tick ( ✓ ) the correct answer.

**4.** Write the following sets in the set builder form.

(i) A = {5, 6, 7, 8} (ii) B = {0, 1, 2}

(iii) C = {*a, e, i, o, u*}

(iv) D = set of natural numbers greater than 100

(v) E = set of odd numbers greater than 1 and less than 10

**5.** Write the following sets in descriptive and tabular form.

(i) $A = \{x \mid x \in W \wedge x < 7\}$

(ii) $B = \{x \mid x \in E \wedge 3 < x < 12\}$

(iii) $C = \{x \mid x \in Z \wedge -2 < x < +2\}$

(iv) $D = \{x \mid x \in P \wedge x < 15\}$

**6.** If A = {3, 4, 5, 6} and B = {2, 4, 6}, then verify that:

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

**7.** If X = {2, 3, 4, 5} and Y = {1, 3, 5, 7}, then find:

(i) X – Y (ii) Y – X





**8.** If A = {*a, c, e, g*}, B = {*a, b, c, d*} and C = {*b, d, f, h*}, then verify that:

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

**9.** If U = set of whole numbers and N = set of natural numbers, then verify that:

(i) $N' \cup N = U$ (ii) $N' \cap N = \phi$

**10.** If U = {*a, b, c, d, e*}, A = {*a, b, c*} and B = {*b, d, e*}, then show through Venn diagram

(i) A′ (ii) B′ (iii) A ∪ B (iv) A ∩ B

## Summary

- There are three forms to write a set.
  (i) Descriptive form (ii) Tabular form (iii) Set builder form
- Two sets are said to be disjoint if there is no element common between them.
- If A and B are two sets then union of A and B is denoted by $A \cup B$ and intersection of A and B is denoted by $A \cap B$.
- If A and B are two sets then B is said to be subset of A if every element of set B is the element of set A.
- Two sets are called overlapping sets if there is at least one element common between them but none of them is a subset of the other.
- A set which contains all possible elements of a given situation or discussion is called the universal set.

CHAPTER

2

# RATIONAL NUMBERS

Animation 2.1: Rational Numbers
Source & Credit: elearn.punjab

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Define a rational number as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.
- Represent rational numbers on number line.
- Add two or more rational numbers.
- Subtract a rational number from another.
- Find additive inverse of rational numbers.
- Multiply two or more rational numbers.
- Divide a rational number by a non-zero rational number.
- Find multiplicative inverse of a non-zero rational number.
- Find reciprocal of a non-zero rational number.
- Verify commutative property of rational numbers with respect to addition and multiplication.
- Verify associative property of rational numbers with respect to addition and multiplication.
- Verify distributive property of rational numbers with respect to multiplication over addition/ subtraction.
- Compare two rational numbers.
- Arrange rational numbers in ascending or descending order.

# 2.1 Rational Numbers

In previous class, we have learnt that the difference of two counting numbers is not always a natural number. For example,

2 – 4 = –σ2 ...............(i)

1 – 5 = –4 ...............(ii)

In (i) and (ii), we can observe that -2 and - 4 are not natural numbers. This problem gave us the idea of integers. Now in integers, when we multiply an integer by another integer, the result is also an integer. For example,

–1 x 2 = –2........... (iii)

–2 x (–3) = 6...........(iv)





From the above (iii) and (iv), we can notice that -2 and 6 are also integers. But in case of division of integers, we do not always get the same result, i.e. $\frac{3}{2}, \frac{4}{7}, -\frac{2}{5}, \frac{1}{6}$ are not integers. So, it means that the division of integers also demands another number system consisting of fractions, as well as, integers that is fulfilled by the rational numbers.

### 2.1.1 Defining Rational Numbers

A number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q m 0, is called a rational number, e.g., $\frac{3}{2}, \frac{4}{7}, -\frac{2}{5}, \frac{1}{6}$ are examples of rational numbers.

| |
|---|
| The set of rational numbers is the set whose elements are natural numbers, negative numbers, zero and all positive and negative fractions. |

### 2.1.2 Representation of Rational Numbers on Number line

We already know the method of constructing a number line to represent the integers. Now we use the same number line to represent the rational numbers. For this purpose, we draw a number line as given below.

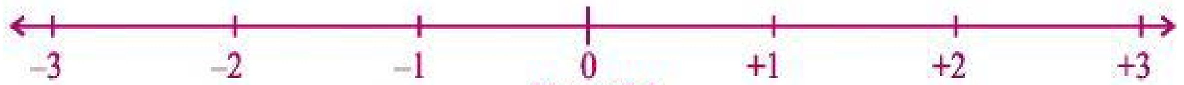



Figure 2.1

Now we divide each segment of the above number line into two equal parts, as given in the following diagram.

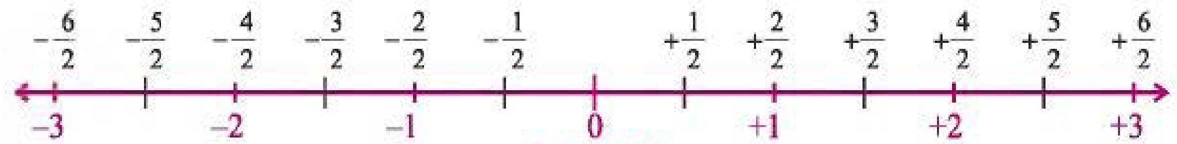



Figure 2.2

In the figure 2.2, the number line represents the rational numbers which are given below.

$$..., -\frac{6}{2}, -\frac{5}{2}, -\frac{4}{2}, -\frac{3}{2}, -\frac{2}{2}, -\frac{1}{2}, 0, +\frac{1}{2}, +\frac{2}{2}, +\frac{3}{2}, +\frac{4}{2}, +\frac{5}{2}, +\frac{6}{2}, ...$$

Now we divide further each small segment of the above drawn number line into two more equal parts.

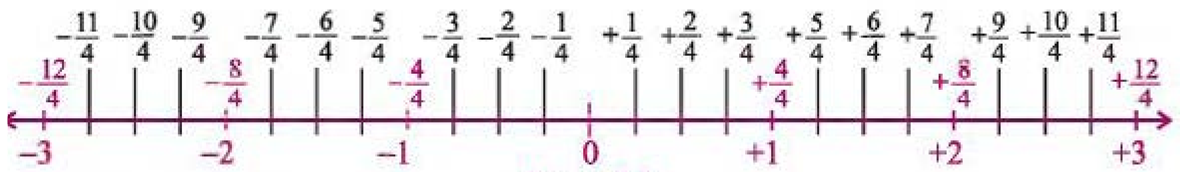



**Figure 2.3**

In the figure 2.3, the number line represents the following rational numbers.

$..., -\frac{12}{4}, -\frac{11}{4}, -\frac{10}{4}, -\frac{9}{4}, -\frac{8}{4}, -\frac{7}{4}, -\frac{6}{4}, -\frac{5}{4}, -\frac{4}{4}, -\frac{3}{4}, -\frac{2}{4}, -\frac{1}{4}, 0, +\frac{1}{4}, +\frac{2}{4}, +\frac{3}{4}, +\frac{4}{4}, +\frac{5}{4}, +\frac{6}{4}, +\frac{7}{4}, +\frac{8}{4}, +\frac{9}{4}, +\frac{10}{4}, +\frac{11}{4}, +\frac{12}{4}, ...$

Similarly, we can divide each segment of a number line into three, five and even more equal parts and we can also represent any rational number on a number line by using the above given method.

**Example 1:** Draw a number line and represent the rational number $\frac{-10}{3}$

**Solution:**

**Step 1:** Draw a number line as given below.


**Step 2:** Convert $\frac{-10}{3}$ to mixed fraction $-3\frac{1}{3}$

**Step 3:** Divide the line segment of the number line between – 4 and –3 in three equal parts and start counting from the point –3 to –4 on the first part is $-3\frac{1}{3}$ which is our required number.






## EXERCISE 2.1

1. Write "T" for a true and "F" for a false statement.
   - (i) Positive numbers are rational numbers.
   - (ii) "0" is not a rational number.
   - (iii) An integer is expressed in $\frac{p}{q}$, form.
   - (iv) Negative numbers are not rational numbers.
   - (v) In any rational number $\frac{p}{q}$, q can be zero.

2. Represent each rational number on the number line.

   (i) $\frac{-5}{2}$ (ii) $\frac{2}{3}$ (iii) $1\frac{4}{5}$ (iv) $-2\frac{3}{4}$

## 2.2 Operations on Rational Numbers

In this section, we perform operation of addition, subtraction, multiplication and division on rational numbers.

### 2.2.1 Addition of Rational Numbers

(a) If $\frac{p}{s}$ and $\frac{q}{s}$ are any two rational numbers with the same denominators, then we shall add them as given below.

$$\frac{p}{s}+\frac{q}{s}=\frac{p+q}{s}$$

**Example 1:** Simplify the following rational numbers.

(i) $\frac{2}{3}+\frac{1}{3}$ (ii) $-\frac{1}{7}+\frac{2}{7}+\frac{4}{7}$ (iii) $\frac{11}{15}+\frac{8}{15}+\left(\frac{-14}{15}\right)$ (iv) $\frac{a}{b}+\frac{c}{b}$

**Solution:**

(i) $\frac{2}{3}+\frac{1}{3}$

$= \frac{2+1}{3}$

$= \frac{3}{3}=1$

(ii) $-\frac{1}{7}+\frac{2}{7}+\frac{4}{7}$ (iii) $\frac{11}{15}+\frac{8}{15}+\left(\frac{-14}{15}\right)$ (iv) $\frac{a}{b}+\frac{c}{b}$

$= \frac{-1+2+4}{7}$ $\quad = \frac{11+8-14}{15}$ $\quad = \frac{a+c}{b}$

$= \frac{5}{7}$ $\quad = \frac{5}{15}=\frac{1}{3}$

(b) If $\frac{p}{q}$, and $\frac{r}{s}$ are any two rational numbers, where $q, s \neq 0$, whose denominators are different, then we can add them by the following formula.

$$\frac{p}{q}+\frac{r}{s}=\frac{ps+rq}{qs}$$

**Example 2:** Find the sum of the following rational numbers.

(i) $-\frac{6}{5}+\frac{7}{12}$ (ii) $1\frac{1}{3}+\frac{5}{2}+\frac{1}{4}$

**Solution:**

(i) $-\frac{6}{5}+\frac{7}{12}$

$= \frac{-72+35}{60}=-\frac{37}{60}$

(ii) $1\frac{1}{3}+\frac{5}{2}+\frac{1}{4}$

$= \frac{4}{3}+\frac{5}{2}+\frac{1}{4}$

$= \frac{16+30+3}{12}=\frac{49}{12}=4\frac{1}{12}$

## 2.2.2 Subtraction of Rational Numbers

(a) Consider tow rational numbers with the same denominators. The difference $\frac{p}{s}-\frac{q}{s}$ is as under:

$$\frac{p}{s}-\frac{q}{s}=\frac{p-q}{s}$$

**Example 3:** Simplify the following.

(i) $\frac{1}{5}-\frac{2}{5}$ (ii) $\frac{8}{9}-\frac{1}{9}-\left(\frac{-4}{9}\right)$





**Solution:**

(i) $\frac{1}{5}-\frac{2}{5}$

$= \frac{1-2}{5}=-\frac{1}{5}$

(ii) $\frac{8}{9}-\frac{1}{9}-\left(\frac{-4}{9}\right)$

$= \frac{8}{9}-\frac{1}{9}+\frac{4}{9}$

$= \frac{8-1+4}{9}=\frac{11}{9}=1\frac{2}{9}$

(b) Consider two rational numbers $\frac{p}{q}, \frac{r}{s}$ with different denominators. The different $\frac{p}{q}-\frac{r}{s}$ is as under:

$$\frac{p}{q}-\frac{r}{s}=\frac{ps-rq}{qs}$$

**Example 4:** Simplify.

(i) $\frac{4}{3}-\left(-\frac{2}{9}\right)$ (ii) $\frac{5}{2}-\frac{3}{4}-\left(-\frac{1}{8}\right)$

**Solution:**

(i) $\frac{4}{3}-\left(-\frac{2}{9}\right)$

$= \frac{4}{3}+\frac{2}{9}$

$= \frac{12+2}{9}=\frac{14}{9}=1\frac{5}{9}$

(ii) $\frac{5}{2}-\frac{3}{4}-\left(-\frac{1}{8}\right)$

$= \frac{5}{2}-\frac{3}{4}+\frac{1}{8}$

$= \frac{20-6+1}{8}=\frac{15}{8}=1\frac{7}{8}$

### 2.2.3 Additive Inverse

Consider that $\frac{p}{q}$ and $\frac{-p}{q}$ are any two rational numbers, then we can add them by the following method.

$$\frac{p}{q}+\left(\frac{-p}{q}\right)=\left(\frac{-p}{q}\right)+\frac{p}{q}=0$$

We can examine that the sum of these two rational numbers is zero. Hence, two rational numbers $\frac{p}{q}$ and $\frac{-p}{q}$ are called additive inverse of each other and 0 is known as additive identity. For example, $\frac{1}{2}$ and $-\frac{1}{2}$, 3 and –3, $\frac{-5}{11}$ and $\frac{5}{11}$, etc. all are additive inverse of each other.

**Example 5:** Write the additive inverse of the following rational numbers.

(i) 3 (ii) $-\frac{1}{2}$ (iii) $\frac{7}{4}$

**Solution:**

(i) To find the additive inverse of 3, change its sign.
Additive inverse of 3 is –3
**Check:** 3 + (–3) = 3 – 3 = 0

(ii) To find the additive inverse of $-\frac{1}{2}$ change its sign.

Additive inverse of $\frac{-1}{2}$ is $\frac{1}{2}$
**Check:** $\frac{-1}{2}+\frac{1}{2}=0$

(iii) To find the additive inverse of $\frac{7}{4}$ change its sign.





Additive inverse of $\frac{7}{4}$ is $\frac{-7}{4}$
**Check:** $\frac{7}{4}+\left(\frac{-7}{4}\right)=\frac{7}{4}-\frac{7}{4}=0$

### 2.2.4 Multiplication of Rational Numbers

We can find the product of two or more rational numbers by the given rule.
**Rule:** Multiply the numerator of one rational number by the numerator of the other rational number. Similarly, multiply the denominators of both rational numbers, i.e.

$$\frac{p}{q}\times\frac{r}{s}=\frac{pr}{qs}$$

**Example 6:** Find the product of the following rational numbers.

(i) $\frac{2}{5}\times\frac{11}{12}$ (ii) $\frac{1}{4}\times\left(-\frac{2}{3}\right)\times\left(-\frac{5}{2}\right)$

**Solution:**

(i) $\frac{2}{5}\times\frac{11}{12}$
$=\frac{2\times11}{5\times12}=\frac{22}{60}=\frac{11}{30}$

(ii) $\frac{1}{4}\times\left(-\frac{2}{3}\right)\times\left(-\frac{5}{2}\right)$
$=\frac{1\times(-2)\times(-5)}{4\times3\times2}=\frac{10}{24}=\frac{5}{12}$

### 2.2.5 Multiplicative Inverse

Consider two rational numbers $\frac{p}{q}$ and $\frac{q}{p}$ where $p \neq 0$ and $q \neq 0$. We find their product by the following formula as under

$$\frac{p}{q}\times\frac{q}{p}=\frac{pq}{pq}=1$$

We can notice that the product of these two rational numbers is 1. Hence, two rational numbers $\frac{p}{q}$ and $\frac{q}{p}$ are known as multiplicative inverse of each other and 1 is called the multiplicative identity. For example, 2 and $\frac{1}{2}$, –5 and $-\frac{1}{5}$, $\frac{3}{7}$ and $\frac{7}{3}$ etc. all are multiplicative inverse of each other.

**Example 7:** Find the multiplicative inverse of the following rational numbers.

(i) $-4$ (ii) $\frac{3}{5}$ (iii) $-\frac{11}{9}$

**Solution:**

(i) $-4$

To find the multiplicative inverse of $-4$, write the numerator as denominator and denominator as numerator.

Multiplicative inverse of $-4$ is $-\frac{1}{4}$

**Check:** $(-4)\times\left(-\frac{1}{4}\right)=1$

(ii) $\frac{3}{5}$

Multiplicative inverse of $\frac{3}{5}$ is $\frac{5}{3}$

**Check:** $\frac{3}{5}\times\frac{5}{3}=\frac{15}{15}=1$

(iii) $-\frac{11}{9}$

Multiplicative inverse of $-\frac{11}{9}$ is $-\frac{9}{11}$

**Check:** $-\frac{11}{9}\times-\frac{9}{11}=\frac{99}{99}=1$

- For any non-zero rational number $\frac{p}{q}$, the rational number $\frac{q}{p}$ is called its reciprocal.
- The number 0 has no reciprocal.
- The multiplicative inverse of a non-zero rational number is its reciprocal.

## 2.2.6 Division of Rational Numbers

We know that division is an inverse operation of multiplication. So, we can do the process of division in the following steps.

**Step 1:** Find the multiplicative inverse of divisor.





**Step 2:** Multiply it by the dividend, according to the rule of multiplication, i.e.

$$\frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\times\frac{s}{r}=\frac{ps}{qr}$$

**Example 8:** Simplify:

(i) $-\frac{8}{3}\div\frac{16}{3}$ (ii) $-\frac{4}{5}\div\left(-\frac{6}{25}\right)$ (iii) $\frac{3}{5}\div\left(-\frac{6}{5}\right)$

**Solution:**

(i) $-\frac{8}{3}\div\frac{16}{3}$

$=-\frac{8}{3}\times\frac{3}{16}=-\frac{1}{2}$

(ii) $-\frac{4}{5}\div\left(-\frac{6}{25}\right)$

$=-\frac{4}{5}\times\left(-\frac{25}{6}\right)$

$=\frac{(-4)\times(-25)}{5\times6}$

$=\frac{(-2)\times(-5)}{3}$

$=\frac{10}{3}$

(iii) $\frac{3}{5}\div\left(-\frac{6}{5}\right)$

$=\frac{3}{5}\times\left(-\frac{5}{6}\right)$

$=\frac{3\times(-5)}{5\times6}=\frac{-1}{2}$

## 2.2.7 Finding Reciprocal of a Rational Number

Consider a non-zero rational number $\frac{3}{7}$ which is made up of two integers 3, as numerator and 7 as denominator. If we interchange the integers in numerator and denominator, we get another rational number $\frac{7}{3}$. In general for any non-zero rational number $\frac{p}{q}$, we have another non-zero rational number $\frac{q}{p}$. This number is called the reciprocal of $\frac{p}{q}$. The number $\frac{7}{3}$ is the reciprocal of $\frac{3}{7}$. Likewise, $\frac{9}{-13}$ or $\frac{-9}{13}$ is the reciprocal of $\frac{-13}{9}$, and $\frac{-105}{113}$ is the reciprocal of $\frac{113}{-105}$ or $\frac{-113}{105}$.

We observe from here that if $\frac{q}{p}$ is the reciprocal of $\frac{p}{q}$, then $\frac{p}{q}$, is the reciprocal of $\frac{q}{p}$. In other words, $\frac{p}{q}$, and $\frac{q}{p}$ are reciprocals of each other.

## EXERCISE 2.2

**1.** Find the additive inverse and multiplicative inverse of the following rational numbers.

(i) $-7$ (ii) $23$ (iii) $-11$ (iv) $\frac{1}{3}$

(v) $\frac{-2}{7}$ (vi) $6$ (vii) $1$ (viii) $\frac{-6}{13}$

(ix) $\frac{1}{100}$ (x) $\frac{18}{27}$ (xi) $-\frac{99}{100}$ (xii) $\frac{102}{117}$

**2.** Simplify the following.

(i) $\frac{1}{8}-\left(-\frac{5}{8}\right)$ (ii) $-\frac{99}{100}+\frac{77}{100}$ (iii) $\frac{3}{4}+\frac{4}{3}$

(iv) $\frac{1}{5}-\frac{3}{20}$ (v) $1+\left(-\frac{49}{50}\right)$ (vi) $1+\frac{11}{100}$

(vii) $\frac{1}{11}+\left(\frac{-5}{11}\right)+\frac{10}{11}$ (viii) $\frac{13}{23}-\frac{10}{23}+\frac{4}{23}$ (ix) $\left(-\frac{1}{2}\right)+\left(-\frac{1}{5}\right)+\frac{9}{10}$

(x) $\frac{1}{8}+\frac{1}{9}-\frac{15}{18}$ (xi) $-\frac{3}{4}-\frac{5}{6}-\left(-\frac{17}{8}\right)$ (xii) $\frac{1}{11}+\frac{11}{10}+\left(-\frac{22}{5}\right)$

**3.** Simplify:

(i) $\frac{8}{9}\times\frac{3}{4}$ (ii) $\frac{50}{51}\times\frac{7}{10}$ (iii) $\frac{121}{169}\div\frac{11}{13}$

(iv) $\frac{5}{7}\div\frac{35}{40}$ (v) $\left(-\frac{15}{28}\right)\times\frac{14}{30}$ (vi) $\frac{111}{100}\div\frac{222}{300}$

(vii) $\frac{3}{2}\div\frac{4}{9}\times\frac{16}{81}$ (viii) $\frac{8}{9}\div\frac{2}{3}\times\frac{15}{28}$ (ix) $\frac{8}{125}\div\frac{16}{75}$

(x) $\frac{1}{5}\times\left(-\frac{2}{5}\right)\times\left(\frac{-100}{32}\right)$ (xi) $\frac{1}{1000}\div\left(-\frac{1}{100}\right)$ (xii) $\frac{-1}{2}\times\frac{3}{5}\div\left(\frac{-51}{40}\right)$

- **Properties of Rational Numbers**

The rational numbers also obey commutative, associative and distributive properties like whole numbers, fractions, integers, etc. Let us verify it with examples.





### 2.2.8 Commutative Property

- **Commutative Property of Rational Numbers w.r.t Addition**

Consider that $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers, then according to the commutative property of addition, we have:

$$\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q}$$

**Example 1:** Prove that $\frac{1}{2}+\frac{2}{3}=\frac{2}{3}+\frac{1}{2}$

**Solution:** $\frac{1}{2}+\frac{2}{3}=\frac{2}{3}+\frac{1}{2}$

L.H.S. $=\frac{1}{2}+\frac{2}{3}$

$=\frac{3+4}{6}=\frac{7}{6}$

R.H.S $=\frac{2}{3}+\frac{1}{2}$

$=\frac{4+3}{6}=\frac{7}{6}$

L.H.S = R.H.S

- **Commutative Property of Rational Numbers w.r.t Multiplication**

According to commutative property of multiplication, for any two rational numbers $\frac{p}{q}$, and $\frac{r}{s}$ we have:

$$\frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$$

**Example 2:** Prove that $\left(-\frac{2}{5}\right)\times\frac{1}{4}=\frac{1}{4}\times\left(-\frac{2}{5}\right)$

**Solution:** $\left(-\frac{2}{5}\right)\times\frac{1}{4}=\frac{1}{4}\times\left(-\frac{2}{5}\right)$

L.H.S $=\left(-\frac{2}{5}\right)\times\frac{1}{4}$

$=-\frac{2}{20}=-\frac{1}{10}$

R.H.S $=\frac{1}{4}\times\left(-\frac{2}{5}\right)$

$=-\frac{2}{20}=-\frac{1}{10}$

L.H.S = R.H.S

**Result:** Commutative property with respect to addition and multiplication holds true for rational numbers.

## 2.2.9 Associative Property

- **Associative Property of Rational Numbers w.r.t Addition**

Consider that $\frac{p}{q}, \frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to the associative property of addition, we have:

$$\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{t}{u}=\frac{p}{q}+\left(\frac{r}{s}+\frac{t}{u}\right)$$

**Example 3:** Prove that $\left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{5}=\frac{1}{4}+\left(\frac{1}{2}+\frac{1}{5}\right)$

**Solution:** $\left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{5}=\frac{1}{4}+\left(\frac{1}{2}+\frac{1}{5}\right)$

$$\text{L.H.S} = \left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{5} = \left(\frac{1+2}{4}\right)+\frac{1}{5} = \frac{3}{4}+\frac{1}{5} = \frac{15+4}{20}=\frac{19}{20}$$

$$\text{R.H.S} = \frac{1}{4}+\left(\frac{1}{2}+\frac{1}{5}\right) = \frac{1}{4}+\left(\frac{5+2}{10}\right) = \frac{1}{4}+\frac{7}{10} = \frac{5+14}{20}=\frac{19}{20}$$

L.H.S = R.H.S

- **Associative Property of Rational Numbers w.r.t Multiplication**

According to associative property of multiplication, for any three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ we have:

$$\left(\frac{p}{q}\times\frac{r}{s}\right)\times\frac{t}{u}=\frac{p}{q}\times\left(\frac{r}{s}\times\frac{t}{u}\right)$$

**Example 4:** Prove that $\left(-\frac{2}{3}\times\frac{1}{2}\right)\times\frac{-3}{4}=-\frac{2}{3}\times\left(\frac{1}{2}\times\frac{-3}{4}\right)$

**Solution:** $\left(-\frac{2}{3}\times\frac{1}{2}\right)\times\frac{-3}{4}=-\frac{2}{3}\times\left(\frac{1}{2}\times\frac{-3}{4}\right)$





$$\text{L.H.S} = \left(-\frac{2}{3}\times\frac{1}{2}\right)\times\frac{-3}{4} = -\frac{1}{3}\times-\frac{3}{4}=\frac{1}{4}$$

$$\text{R.H.S} = -\frac{2}{3}\times\left(\frac{1}{2}\times\frac{-3}{4}\right) = -\frac{2}{3}\times-\frac{3}{8}=\frac{1}{4}$$

L.H.S = R.H.S

**Result:** Associative property with respect to addition and multiplication holds true for rational numbers.

## 2.2.10 Distributive Property of Multiplication over Addition and Subtraction

Now again consider the three rational numbers $\frac{p}{q}, \frac{r}{s}$ and $\frac{t}{u}$ then according to the distributive property:

(i) $\frac{p}{q}\times\left(\frac{r}{s}+\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)+\left(\frac{p}{q}\times\frac{t}{u}\right)$ (ii) $\frac{p}{q}\times\left(\frac{r}{s}-\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)-\left(\frac{p}{q}\times\frac{t}{u}\right)$

**Example 5:** Prove that

(i) $\frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right)=\left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right)$ (ii) $\frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}\right)=\left(\frac{1}{4}\times\frac{1}{2}\right)-\left(\frac{1}{4}\times\frac{1}{6}\right)$

**Solution:**

(i) $\frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right)=\left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right)$

$$\text{L.H.S.} = \frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right) = \frac{1}{5}\times\left(\frac{9+5}{10}\right) = \frac{1}{5}\times\frac{14}{10}=\frac{7}{25}$$

$$\text{R.H.S} = \left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right) = \frac{9}{50}+\frac{1}{10} = \frac{9+5}{50}=\frac{14}{50}=\frac{7}{25}$$

L.H.S = R.H.S

(ii) $\frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}\right)=\left(\frac{1}{4}\times\frac{1}{2}\right)-\left(\frac{1}{4}\times\frac{1}{6}\right)$

$$\text{L.H.S} = \frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}\right) = \frac{1}{4}\times\left(\frac{3-1}{6}\right) = \frac{1}{4}\times\frac{2}{6}=\frac{1}{12}$$

$$\text{R.H.S} = \left(\frac{1}{4}\times\frac{1}{2}\right)-\left(\frac{1}{4}\times\frac{1}{6}\right) = \frac{1}{8}-\frac{1}{24} = \frac{3-1}{24}=\frac{2}{24}=\frac{1}{12}$$

$$\text{L.H.S} = \text{R.H.S}$$

## 2.3.11 Comparison of Rational Numbers

We have studied the comparison of integers and fractions in our previous class. Similarly, we can compare the rational numbers by using the same rules for comparison. We shall make it clear with examples.

- **Case I: Same Denominators**

**Example 6:** Compare the following pairs of rational numbers.

(i) $\frac{2}{7},\frac{4}{7}$ (ii) $\frac{-1}{6},\frac{-5}{6}$ (iii) $\frac{1}{4},\frac{-3}{4}$

**Solution:**

(i) $\frac{2}{7},\frac{4}{7}$

It can be seen that:

$2<4$

So, $\frac{2}{7}<\frac{4}{7}$

(ii) $\frac{-1}{6},\frac{-5}{6}$

It can be seen that:

$-1>-5$

So, $\frac{-1}{6}>\frac{-5}{6}$

(iii) $\frac{1}{4},\frac{-3}{4}$

It can be seen that:

$1>-3$

So, $\frac{1}{4}>\frac{-3}{4}$





- **Case II: Different Denominators**

**Example 7:** Put the correct sign > or < between the following pairs of rational numbers.

(i) $\frac{1}{2},\frac{3}{5}$ (ii) $\frac{9}{-11},\frac{-41}{121}$

**Solution:**

(i) $\frac{1}{2},\frac{3}{5}$

Write other two rational numbers from the given rational numbers such that their denominators must be equal.

$\frac{1}{2}=\frac{1\times5}{2\times5}=\frac{5}{10}$ $\frac{3}{5}=\frac{3\times2}{5\times2}=\frac{6}{10}$

Now compare the numerators of rational numbers with the same denominators.

$5 < 6$

$\frac{5}{10}<\frac{6}{10}$

Thus, $\frac{1}{2}<\frac{3}{5}$

(ii) $\frac{9}{-11},\frac{-41}{121}$

By making their denominators equal

$\frac{9}{-11}=\frac{9\times(-11)}{-11\times(-11)}=\frac{-99}{121}$

Now compare the numerators of rational numbers with the same denominators.

$-99<-41$

$\frac{-99}{121}<\frac{-41}{121}$

Thus, $\frac{9}{-11}<\frac{-41}{121}$

## 2.3.12 Arranging Rational Numbers in Orders

We can also arrange the given rational numbers in ascending order (lowest to highest) and in descending order (highest to lowest) in the following steps.

**Step 1:** Find the L.C.M of the denominators of given rational numbers.
**Step 2:** Rewrite the rational numbers with a common denominator.
**Step 3:** Compare the numerators and arrange the rational numbers in ascending or descending order.

**Example 8:** Arrange the rational numbers $\frac{1}{2},\frac{2}{3}$ and $\frac{7}{8}$ in descending order.

**Solution:**

**Step 1:** The L.C.M of denominators 2, 3 and 8 is 24.
**Step 2:** Rewrite the rational numbers with a common denominator as,

$\frac{1}{2}=\frac{1\times12}{2\times12}=\frac{12}{24}$ $\quad$ $\frac{2}{3}=\frac{2\times8}{3\times8}=\frac{16}{24}$ $\quad$ $\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$

**Step 3:** Compare the numerators 12, 16 and 21 and rearrange the rational numbers in descending order.

$$21 > 16 > 12$$

$$\frac{21}{24}>\frac{16}{24}>\frac{12}{24} \text{ or } \frac{7}{8}>\frac{2}{3}>\frac{1}{2}$$

Thus, arranging in descending order, we get $\frac{7}{8},\frac{2}{3},\frac{1}{2}$.

**Example 9:** Arrange the rational numbers $\frac{1}{4},\frac{2}{3}$ and $\frac{1}{12}$ in ascending order.

**Solution:**

**Step 1:** The L.C.M of denominators 4, 3 and 12 is 12.
**Step 2:** Rewrite the rational numbers with a common denominator as,

$\frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12}$ $\quad$ $\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$ $\quad$ $\frac{1}{12}=\frac{1\times1}{12\times1}=\frac{1}{12}$





**Step 3:** Compare the numerators 3, 8 and 1 and rearrange the rational numbers in ascending order.

$$1 < 3 < 8$$

$$\frac{1}{12}<\frac{3}{12}<\frac{8}{12} \text{ or } \frac{1}{12}<\frac{1}{4}<\frac{2}{3}$$

Thus, arranging in ascending order, we get $\frac{1}{12},\frac{1}{4},\frac{2}{3}$.

## EXERCISE 2.3

**1.** Put the correct sign > , < or = between the following pairs of rational numbers.

(i) $\frac{1}{2},\frac{15}{20}$ (ii) $\frac{2}{-3},\frac{1}{6}$ (iii) $\frac{-1}{5},\frac{2}{-10}$ (iv) $\frac{-1}{9},\frac{-4}{3}$
(v) $-1, \frac{-2}{3}$ (vi) $\frac{1}{2},1$ (vii) $\frac{5}{7},\frac{-1}{2}$ (viii) $\frac{11}{-10},\frac{-10}{11}$
(ix) $\frac{4}{-100},\frac{-1}{25}$ (x) $\frac{-4}{7},\frac{5}{-2}$ (xi) $\frac{4}{9},\frac{6}{-7}$ (xii) $\frac{-8}{11},\frac{3}{-10}$

**2.** Arrange the following rational numbers in descending order.

(i) $\frac{1}{2},\frac{2}{3},\frac{8}{9}$ (ii) $\frac{1}{6},\frac{3}{4},\frac{1}{2}$ (iii) $\frac{4}{7},\frac{1}{3},\frac{5}{6}$

**3.** Arrange the following rational numbers in ascending order.

(i) $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ (ii) $\frac{4}{5},\frac{1}{10},\frac{2}{15}$ (iii) $\frac{3}{8},\frac{1}{4},\frac{5}{6}$

**4.** Prove that:

(i) $\left(\frac{-1}{2}\right)+\frac{1}{3}=\frac{1}{3}+\left(-\frac{1}{2}\right)$ (ii) $\frac{10}{11}+\left(\frac{5}{-44}\right)=\left(\frac{5}{-44}\right)+\frac{10}{11}$

(iii) $\left(\frac{12}{-105}\right)\times\left(\frac{-15}{84}\right)=\left(\frac{-15}{84}\right)\times\left(\frac{12}{-105}\right)$ (iv) $-\frac{2}{3}\times\left(\frac{7}{8}\times\frac{9}{14}\right)=\left(-\frac{2}{3}\times\frac{7}{8}\right)\times\frac{9}{14}$

(v) $\frac{3}{5}+\left(\frac{1}{2}+\frac{7}{10}\right)=\left(\frac{3}{5}+\frac{1}{2}\right)+\frac{7}{10}$ (vi) $\frac{1}{-2}+\left(\frac{3}{5}+\frac{1}{4}\right)=\left(\frac{1}{-2}+\frac{3}{5}\right)+\frac{1}{4}$

(vii) $\frac{2}{3}\times\left(\frac{1}{2}+\frac{5}{6}\right)=\left(\frac{2}{3}\times\frac{1}{2}\right)+\left(\frac{2}{3}\times\frac{5}{6}\right)$ (viii) $\frac{1}{4}\times\left(\frac{8}{9}-\frac{12}{15}\right)=\left(\frac{1}{4}\times\frac{8}{9}\right)-\left(\frac{1}{4}\times\frac{12}{15}\right)$

(ix) $\frac{-5}{8}\times\left(\frac{4}{7}-\frac{2}{3}\right)=\left(\frac{-5}{8}\times\frac{4}{7}\right)-\left(\frac{-5}{8}\times\frac{2}{3}\right)$ (x) $\frac{24}{49}\times\left(\frac{7}{8}+\frac{14}{6}\right)=\left(\frac{24}{49}\times\frac{7}{8}\right)+\left(\frac{24}{49}\times\frac{14}{6}\right)$

## REVIEW EXERCISE 2

**1.** Answer the following questions.

(i) Define a rational number.

(ii) Write the additive inverse of the rational numbers "a".

(iii) What is the reciprocal of the rational number $\frac{p}{q}$, $q \neq 0$?

(iv) Write the sum of two rational numbers $\frac{p}{q}$, and $\frac{r}{s}$ $q$, $r \neq 0$?

(v) What is the rule to find the product of two rational numbers?

(vi) What are the inverse operations of addition and multiplication?

**2.** Fill in the blanks.

(i) The_______ consists of fractions as well as integers.

(ii) The rational numbers $\frac{p}{q}$, and $\frac{-p}{q}$ are called____ inverse of each other.

(iii) A number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q $\neq$? 0 is called the_______ number.

(iv) 0 is called additive identity whereas 1 is called _______ identity.

(v) The rational number 0 has no _______ .

(vi) The _______ inverse of a rational number is its reciprocal.

**3.** Tick ( ✓ ) the correct answer.

**4.** Draw the number lines and represent the following rational numbers.

(i) $1\frac{1}{2}$ (ii) $3\frac{1}{3}$ (iii) $-\frac{1}{4}$ (iv) $-1\frac{4}{5}$





**5.** Find the additive and multiplicative inverse of the following rational numbers.

(i) −14 (ii) $\frac{1}{5}$ (iii) $-\frac{2}{3}$ (iv) $-\frac{11}{27}$

**6.** Put the correct sign > or < between the following pairs of rational numbers.

(i) $\frac{1}{4}, \frac{-1}{2}$ (ii) $\frac{2}{3}, \frac{1}{5}$ (iii) $\frac{-11}{17}, \frac{3}{8}$

(iv) $\frac{10}{13}, \frac{11}{14}$ (v) $\frac{-4}{9}, \frac{2}{-5}$ (vi) $\frac{5}{-22}, \frac{-11}{25}$

**7.** Solve the following.

(i) $\left(\frac{-19}{55}\right)+\frac{51}{55}+\left(\frac{-21}{55}\right)$ (ii) $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$ (iii) $\left(\frac{-1}{3}\right)+\left(\frac{-1}{4}\right)+\frac{1}{2}$

(iv) $\frac{2}{7}-\frac{1}{2}+\frac{3}{14}$ (v) $\frac{5}{8}+\frac{1}{5}-\frac{3}{4}$ (vi) $\left(\frac{-11}{15}\right)+\left(\frac{-3}{5}\right)+\frac{5}{4}$

**8.** Simplify the following.

(i) $\frac{2}{3}\div\frac{16}{21}\times\frac{27}{49}$ (ii) $\left(\frac{-1}{100}\right)\div\left(\frac{1}{10}\right)$ (iii) $\frac{1}{5}\times\frac{2}{3}\times\left(\frac{-30}{44}\right)$

(iv) $\frac{1}{6}\times\left(\frac{-2}{3}\right)\div\left(\frac{-11}{63}\right)$ (v) $\frac{-2}{7}+\frac{3}{4}\times\frac{63}{100}$ (vi) $\frac{8}{21}\div\frac{7}{12}$

**9.** Prove that:

(i) $(-1)+\frac{35}{54}=\frac{35}{54}+(-1)$ (ii) $\frac{-4}{5}\times\left(\frac{1}{8}+\frac{11}{12}\right)=\left(\frac{-4}{5}\times\frac{1}{8}\right)+\left(\frac{-4}{5}\times\frac{11}{12}\right)$

(iii) $\frac{4}{9}\times\left(\frac{2}{3}\times\frac{5}{7}\right)=\left(\frac{4}{9}\times\frac{2}{3}\right)\times\frac{5}{7}$ (iv) $\left(\frac{-121}{169}\right)\times\left(\frac{13}{-11}\right)=\left(\frac{13}{-11}\right)\times\left(\frac{-121}{169}\right)$

(v) $\frac{-1}{4}+\left(\frac{1}{6}+\frac{3}{5}\right)=\left(\frac{-1}{4}+\frac{1}{6}\right)+\frac{3}{5}$ (vi) $\frac{5}{12}\times\left(\frac{-2}{7}-2\right)=\left(\frac{5}{12}\times\frac{-2}{7}\right)-\left(\frac{5}{12}\times2\right)$

## SUMMARY

- Every integer can be divided by another non-zero integer, the number obtained is called a rational number and is written symbolically as $\frac{p}{q}$.

- Addition of rational numbers with:

Same denominators. Different denominators.

$$\frac{p}{s}+\frac{q}{s}=\frac{p+q}{s} \qquad \frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q}$$

- Subtraction of rational numbers with:

Same denominators. Different denominators.

$$\frac{p}{s}-\frac{q}{s}=\frac{p-q}{s} \qquad \frac{p}{q}-\frac{r}{s}=\frac{ps-rq}{qs}$$

- To find the product of two rational numbers, multiply the numerator of one rational number by the numerator of the other. Similarly, multiply the denominators.

$$\frac{p}{q}\times\frac{r}{s}=\frac{pr}{qs}$$

- Division is an inverse operation of multiplication. So, for any two rational numbers.

$$\frac{p}{q},\frac{r}{s} \qquad \frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\times\frac{s}{r}=\frac{ps}{qr}$$

- 0 is called additive identity and 1 is called multiplicative identity.
- $\frac{q}{p}$ is called the reciprocal of $\frac{p}{q}$,
- If $\frac{p}{q},\frac{r}{s}$ are two rational numbers, then according to the commutative property:

$$\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q} \qquad \frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$$

- If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to the associative property.

$$\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{t}{u}=\frac{p}{q}+\left(\frac{r}{s}+\frac{t}{u}\right) \qquad \left(\frac{p}{q}\times\frac{r}{s}\right)\times\frac{t}{u}=\frac{p}{q}\times\left(\frac{r}{s}\times\frac{t}{u}\right)$$

- Now again consider the three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ then according to the distributive property:

$$\frac{p}{q}\times\left(\frac{r}{s}+\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)+\left(\frac{p}{q}\times\frac{t}{u}\right) \qquad \frac{p}{q}\times\left(\frac{r}{s}-\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)-\left(\frac{p}{q}\times\frac{t}{u}\right)$$

CHAPTER

3

# DECIMALS

Animation 1.1: Introduction to Decimals
Source & Credit: eLearn.Punjab

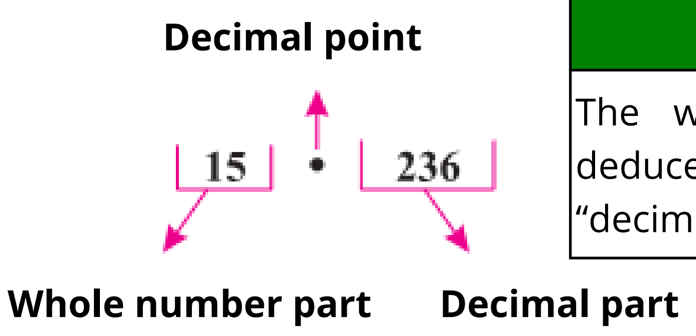
## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Convert decimals to rational numbers.
- Define terminating decimals as decimals having a finite number of digits after the decimal point.
- Define recurring decimals as non-terminating decimals in which a single digit or a block of digits repeats itself an infinite number of times after decimals point (e.g. = 0.285714285714285714....)
- Use the following rule to find whether a given rational number is terminating or not.
- Rule: If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal.
- Express a given rational number as a decimal and indicate whether it is terminating or recurring.
- Get an approximate value of a number, called rounding off, to a desired number of decimal places.

## Introduction

In the previous classes, we have learnt that a decimal consists of two parts, i.e. a whole number part and a decimal part. To separate these parts in a number, we place a dot between them which is known as the decimal point.




**Do you Know**

The word "decimal" has been deduced from a latin word "decimus" which means the tenth.

So, we can define a decimal; a number with a decimal point is called a decimal.





## 3.1 Conversion of Decimals to Rational Numbers

We take the following steps to convert decimals to rational numbers.

**Step 1**: Write "1" below the decimal point.

**Step 2**: Add as many zeros as the digits after the decimal point.

**Step 3**: Reduce the rational number to the lowest form.

**Example 1:** Convert 0.12 to a rational number.

**Solution:**

$$0.12 = \frac{12}{100}$$

$$= \frac{12 \div 4}{100 \div 4} = \frac{3}{25}$$

Thus, $0.12 = \frac{3}{25}$

**Example 2:** Convert 2.55 to a rational number.

**Solution:**

$$2.55 = \frac{255}{100}$$

$$= \frac{255 \div 5}{100 \div 5} = \frac{51}{20}$$

Thus, $2.55 = \frac{51}{20}$

**Example 3:** Convert –1.375 to a rational number.

**Solution:**

$$-1.375 = -\frac{1375}{1000}$$

Find the HCF of 1375 and 1000.

1000 ) 1375 ( 1
−1000
375 ) 1000 ( 2
−750
250 ) 375 ( 1
−250
125 ) 250 ( 2
−250
0

$$= -\frac{1375 \div 125}{1000 \div 125} = -\frac{11}{8}$$

Thus, $-1.375 = -\frac{11}{8}$

## EXERCISE 3.1

1. Convert the following decimals into rational numbers.

| | | |
|---|---|---|
| (i) 0.36 | (ii) 0.75 | (iii) –0.125 |
| (iv) –6.08 | (v) 6.46 | (vi) 15.25 |
| (vii) 8.125 | (viii) –0.00625 | (ix) –0.268 |

# 3.2 Terminating and Non-Terminating Decimals

Decimals can be classified into two classes.
(i) Terminating Decimals (ii) Non-terminating Decimals

## 3.2.1 Terminating Decimals

Look at the conversion of rational numbers $\frac{1}{4}, \frac{2}{5}, \frac{4}{25}$ into decimals.

(i) $\frac{1}{4}$

$$\begin{array}{r|l} & 0.25 \\ \hline 4 & 10 \\ & -8 \\ \hline & 20 \\ & -20 \\ \hline & 0 \end{array}$$

Thus, $\frac{1}{4} = 0.25$

(ii) $\frac{2}{5}$

$$\begin{array}{r|l} & 0.4 \\ \hline 5 & 20 \\ & -20 \\ \hline & 0 \end{array}$$

Thus, $\frac{2}{5} = 0.4$

(iii) $\frac{4}{25}$

$$\begin{array}{r|l} & 0.16 \\ \hline 25 & 40 \\ & -25 \\ \hline & 150 \\ & -150 \\ \hline & 0 \end{array}$$

Thus, $\frac{4}{25} = 0.16$

In the above example, we observe that after a finite number of steps, we obtain a zero as remainder. Such rational numbers, for which long division terminates after a finite number of steps, can be expressed in decimal form with finite decimal places and these decimals are called terminating decimals which can be defined as; "A decimal in which the number of digits after the decimal point is finite, is called a terminating decimal."





**Example 1:** Express each rational number as a decimal.

(i) $\frac{7}{8}$ (ii) $\frac{18}{25}$ (iii) $\frac{627}{625}$

**Solution:**

(i) $\frac{7}{8}$

$$\begin{array}{r|l} & 0.875 \\ \hline 8 & 70 \\ & -64 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -40 \\ \hline & 0 \end{array}$$

Thus, $\frac{7}{8} = 0.875$

(ii) $\frac{18}{25}$

$$\begin{array}{r|l} & 0.72 \\ \hline 25 & 180 \\ & -175 \\ \hline & 50 \\ & -50 \\ \hline & 0 \end{array}$$

Thus, $\frac{18}{25} = 0.72$

(iii) $\frac{627}{625}$

$$\begin{array}{r|l} & 1.0032 \\ \hline 625 & 627 \\ & -625 \\ \hline & 2000 \\ & -1875 \\ \hline & 1250 \\ & -1250 \\ \hline & 0 \end{array}$$

Thus, $\frac{627}{625} = 1.0032$

## 3.2.2 Non-Terminating Decimals

In some cases while converting a rational number into a decimal, division never ends. Such decimals are called non-termination decimals as shown in the following examples.

(i) $\frac{1}{3}$

$$\begin{array}{r} 0.3333... \\ 3\overline{)10} \\ -9 \\ \hline 10 \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \end{array}$$

$\frac{1}{3} = 0.3333...$

(ii) $\frac{3}{11}$

$$\begin{array}{r} 0.2727... \\ 11\overline{)30} \\ -22 \\ \hline 80 \\ -77 \\ \hline 30 \\ -22 \\ \hline 80 \\ -77 \\ \hline 3 \end{array}$$

$\frac{3}{11} = 0.2727...$

(iii) $\frac{1}{6}$

$$\begin{array}{r} 0.1666... \\ 6\overline{)10} \\ -6 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

$\frac{1}{6} = 0.1666...$

So, we can define a non-terminating decimal as;
"A decimal in which the number of digits after the decimal point are infinite, is called a non-terminating decimal".
From the above examples, it can also be observed that a single digit or a block of digits repeats itself an infinite number of times after the decimal point in such decimals. i.e.

- In 0.3333..., the digit 3 repeats itself an infinite number for times.
- In 0.2727..., the block of digits 27 repeats itself an infinite number of times.
- In 0.1666..., the digit 6 repeats itself an infinite number of times.

The non-termination decimals in which a single digit or a block of digits repeats itself infinite number of times after the decimal point are also called recurring decimals.

**Example 2:** Change the rational numbers into non-terminating decimals.

**Solution:**

(i) $\frac{1}{7}$

$$\begin{array}{r} 0.1428571... \\ 7\overline{)10} \\ -7 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 60 \\ -56 \\ \hline 40 \\ -35 \\ \hline 50 \\ -49 \\ \hline 10 \\ -7 \\ \hline 3 \end{array}$$

Thus, $\frac{1}{7} = 0.1428...$

(ii) $-\frac{4}{9}$

$$\begin{array}{r} 0.4444... \\ 9\overline{)40} \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

Thus, $-\frac{4}{9} = -0.4444...$

(iii) $\frac{2}{3}$

$$\begin{array}{r} 0.6666... \\ 3\overline{)20} \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

Thus, $\frac{2}{3} = 0.6666...$





### 3.2.3 Rule to find whether a given rational is terminating or not

We have learnt that the division process terminates for some rational numbers and does not terminate for certain other rational numbers.

- **Terminating Decimals**

$\frac{1}{8} = 0.125$ $\frac{2}{25} = 0.08$ $\frac{7}{4} = 1.75$

- **Non-terminating Decimals**

$\frac{4}{3} = 1.333...$ $\frac{25}{7} = 3.571...$ $\frac{1}{6} = 0.166...$

From the above examples, it can be observed that a rational number can be expressed as a terminating decimal if its denominator has only prime factors 2 and 5, otherwise it is a non-terminating decimal. So, we can use the following rule to find whether the given rational number is terminating or not.

**Rule:** If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal.

**Example 3:** Without actual division, separate terminating and non-terminating decimals.

(i) $\frac{9}{7}$ (ii) $\frac{17}{8}$ (iii) $\frac{20}{6}$ (iv) $\frac{45}{25}$

**Solution:**

(i) $\frac{9}{7}$

$\frac{9}{7}$ is a non-terminating decimal because its denominator is 7.

(ii) $\frac{17}{8}$

$\frac{17}{8}$ is a terminating decimal because its denominator has prime factors 2 x 2 x 2 = 8

(iii) $\frac{20}{6}$

Write in the standard form of the given rational number. $\frac{20}{6}=\frac{20\div 2}{6\div 2}=\frac{10}{3}$

$\frac{20}{6}$ is a non-terminating decimal because the denominator of its standard form is 3.

(iv) $\frac{45}{25}$

The standard form of $\frac{45}{25}=\frac{45\div 5}{25\div 5}=\frac{9}{5}$.

$\frac{45}{25}$ is a terminating decimal because the denominator of its standard form is 5.

### 3.2.4 Expressing a Rational Number as a Decimal to indicate whether it is Terminating or Recurring

**Example 4:** Express the rational numbers as decimals. Also separate terminating and recurring decimals.

(i) $\frac{19}{25}$ (ii) $\frac{17}{45}$ (iii) $\frac{-2}{11}$ (iv) $\frac{-15}{8}$





**Solution:**

(i) $\frac{19}{25}$

$$\begin{array}{r} 0.76 \\ 25\,\overline{)\,190} \\ -175 \\ \hline 150 \\ -150 \\ \hline 0 \end{array}$$

Thus, $\frac{19}{25}=0.76$ which is a terminating decimal.

(ii) $\frac{17}{45}$

$$\begin{array}{r} 0.377... \\ 45\,\overline{)\,170} \\ -135 \\ \hline 350 \\ -315 \\ \hline 350 \\ -315 \\ \hline 35 \end{array}$$

Thus, $\frac{17}{45}=0.377...$ which is a recurring decimal.

(iii) $\frac{-2}{11}$

$$\begin{array}{r} 0.181... \\ 11\,\overline{)\,20} \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \\ -11 \\ \hline 9 \end{array}$$

Thus, $\frac{-2}{11}=-0.181...$ which is a recurring decimal.

(iv) $\frac{-15}{8}$

$$\begin{array}{r} 1.875 \\ 8\,\overline{)\,15} \\ -8 \\ \hline 70 \\ -64 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Thus, $\frac{-15}{8}=-1.875$ which is a terminating decimal.

## 3.3 Approximate Values

Whenever we come across the non-terminating decimals, it is very difficult to solve the problems without the help of a calculator. Even calculators also have limitations. Therefore, in order to solve such kinds of problems, we round off the decimals.

- **Round off**

Here the term round off is used to leave the digits after the decimal point. The following are the steps to round off a decimal.

**Step 1**: Decide how many digits we need after the decimal point.

**Step 2**: Drop the remaining digits off, if the first most digit we want to leave is less than 5. And if it is 5 or more, then add 1 to the

> The symbol "≈" means "approximately equal to".

required last digit before dropping the remaining digits.
It will be easier for us to understand this method with some examples which are given below.

**Example 4:** Round off the following decimals up to:
(a) 3-decimal places
(b) 2-decimal places
(i) 2.3427 (ii) 4.7451 (iii) 1.5349

**Solution:** (i) 2.3427
(a) The digit next to 3-decimal places is 7 (greater than 5). So, we increase the digit 2 by one. i.e. $2.3427 \approx 2.343$
(b) The digit next to 2-decimal places is 2 (less than 5). So, we ignore the remaining digits without any change. i.e. $2.3427 \approx 2.34$
(ii) 4.7451
(a) The digit next to 3-decimal places is 1 (less than 5). So, we ignore the remaining digits without any change. i.e. $4.7451 \approx 4.745$
(b) The digit next to 2-decimal places is 5 (equal to 5). So, we increase the digit 4 by one. i.e. $4.7451 \approx 4.75$
(iii) 1.5349
(a) The digit next to 3-decimal places is 9 (greater than 5). So, we increase the digit 4 by one. i.e. $1.5349 \approx 1.535$
(b) The digit next to 2-decimal places is 4 (less than 5). So, we ignore the remaining digits without any change. i.e. $1.5349 \approx 1.53$

## EXERCISE 3.2

1. Without actual division, separate the terminating and non-terminating decimals.

(i) $\frac{13}{8}$ (ii) $\frac{7}{25}$ (iii) $\frac{8}{3}$ (iv) $\frac{5}{11}$
(v) $\frac{9}{6}$ (vi) $\frac{20}{15}$ (vii) $\frac{22}{7}$ (viii) $\frac{4}{9}$





2. Express the following rational numbers in terminating decimals.

(i) $\frac{2}{100}$ (ii) $\frac{27}{20}$ (iii) $\frac{3}{25}$
(iv) $\frac{31}{50}$ (v) $\frac{5}{1000}$ (vi) $\frac{20}{8}$
(vii) $\frac{21}{6}$ (viii) $\frac{84}{64}$ (ix) $\frac{24}{32}$

3. Express the following rational numbers in non-terminating decimals up to three decimal places.

(i) $\frac{4}{3}$ (ii) $\frac{2}{7}$ (iii) $\frac{5}{11}$ (iv) $\frac{8}{13}$
(v) $\frac{10}{6}$ (vi) $\frac{24}{22}$ (vii) $\frac{7}{12}$ (viii) $\frac{26}{91}$

4. Round off the following decimals up to three decimal places.
(i) 5.41679 (ii) 11.10365 (iii) 0.92517
(iv) 3.10351 (v) 0.74206 (vi) 23.15147

## REVIEW EXERCISE 3

**1. Answer the following questions.**
(i) Define the terminating decimals.
(ii) Write the names of two classes of decimals.
(iii) Which of the non-terminating decimals are called recurring decimal?
(iv) How many digits after a decimal point show a non-terminating decimal?
(v) Write the rule to find whether a given rational number is terminating or not.
(vi) What is meant by the term round off in decimals?

**2. Fill in the blanks.**
(i) A ________ decimal may be recurring or non-recurring.
(ii) Two parts of decimal number separated by a dot is called the ________ .
(iii) In terminating decimals, division ________ after a finite number of steps.

(iv) In decimals, the term round off is used to leave the digits after the ________ .

(v) A fraction will be terminating if the ________ has 2 or 5 or both as factors.

**3. Tick ( ✓ ) the correct answer.**

**4. Convert the following decimals into rational numbers.**

(i) 0.375 (ii) 0.25 (iii) 0.5 (iv) 4.75
(v) 0.79 (vi) 1.29 (vii) 2.34

**5. Convert the following into decimal fractions and identify terminating and non-terminating fractions.**

(i) $\frac{4}{5}$ (ii) $\frac{11}{12}$ (iii) $\frac{8}{9}$ (iv) $\frac{1}{7}$
(v) $\frac{22}{7}$ (vi) $\frac{21}{6}$ (vii) $\frac{3}{10}$

**6. Round off the following up to 2-decimal places.**

(i) 4.5723 (ii) 107.328 (iii) 5.7395
(iv) 6.7982 (v) 25.4893

## SUMMARY

- Every decimal with finite digits after the decimal point is called a terminating decimal.
- A terminating decimal represents a rational number.
- A decimal with infinite digits after a decimal point is called a non-terminating decimal.
- A non terminating decimal may be recurring or non-recurring.
- Decimals can be reduced by rounding off the digits after the decimal point.
- A fraction will be terminating if the denominator in standard form has 2 or 5 or both as factors.

CHAPTER

4

# EXPONENTS

Animation 4.1: Exponents
Source & Credit: elearn.punjab

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Identify base, exponent and value.
- Use rational numbers to deduce laws of exponents.
- **Product Law:**
  when bases are same but exponents are different:
  $$a^m \times a^n = a^{m+n}$$
  when bases are different but exponents are same:
  $$a^n \times b^n = (ab)^n$$
- **Quotient Law:**
  when bases are same but exponents are different:
  $$a^m \div a^n = a^{m-n}$$
  when bases are different but exponents are same:
  $$a^n \div b^n = \left(\frac{a}{b}\right)^n$$
- **Power Law:** $(a^m)^n = a^{mn}$
  For zero exponent: $a^0 = 1$
  For exponent as negative integer: $a^{-m} = \frac{1}{a^m}$
- Demonstrate the concept of power of integer that is $(-a)^n$ when n is even or odd integer.
- Apply laws of exponents to evaluate expressions.

# 4.1 Exponents/Indices

## 4.1.1 Identification of Base, Exponent and Value

We have studied in our previous class that the repeated multiplication of a number can be written in short form, using exponent. For example,

- $7 \times 7 \times 7$ can be written as $7^3$. We read it as 7 to the power of 3 where 7 is the base and 3 is the exponent or index.

> The exponent of a number indicates us, how many times a number (base) is multiplied with itself.





Similarly,

- $11 \times 11$ can be written as $11^2$. We read it as 11 to the power of 2 where 11 is the base and 2 is the exponent.

From the above examples we can conclude that if a number "a" is multiplied with itself n –1 times, then the product will be $a^n$, i.e.

$a^n = a \times a \times a \times a \times \ldots\ldots\ldots\ldots\ldots \times a$ (n-1 times multiplications of "a" with itself)

We read it as "a to the power of n"or "nth power of a"where "a" is the base and "n" is the exponent.

**Example 1:** Express each of the following in exponential form.

(i) $(-3)\times(-3)\times(-3)$ (ii) $2\times2\times2\times2\times2\times2\times2$

(iii) $\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)$ (iv) $\left(\frac{-7}{12}\right)\times\left(\frac{-7}{12}\right)$

**Solution:**

(i) $(-3)\times(-3)\times(-3)=(-3)^3$

(ii) $2\times2\times2\times2\times2\times2\times2=(2)^7$

(iii) $\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)=\left(\frac{1}{4}\right)^4$

(iv) $\left(\frac{-7}{12}\right)\times\left(\frac{-7}{12}\right)=\left(\frac{-7}{12}\right)^2$

**Example 2:** Identify the base and exponent of each number.

(i) $13^{25}$ (ii) $\left(\frac{-7}{11}\right)^9$ (iii) $a^m$ (iv) $(-426)^{11}$ (v) $\left(\frac{a}{b}\right)^n$ (vi) $\left(-\frac{x}{y}\right)^t$

**Solution:**

(i) $13^{25}$
base = 13
exponent = 25

(ii) $\left(-\frac{7}{11}\right)^9$
base = $\frac{-7}{11}$
exponent = 9

(iii) $a^m$
base = $a$
exponent = $m$

(iv) $(-426)^{11}$
base = – 426
exponent = 11

(v) $\left(\frac{a}{b}\right)^n$
base = $\frac{a}{b}$
exponent = $n$

(vi) $\left(\frac{-x}{y}\right)^t$
base = $\frac{-x}{y}$
exponent = $t$

**Example 3:** Write the following in the simplest form.

(i) $(-5)^3$ (ii) $\left(\frac{2}{3}\right)^2$ (iii) $\left(\frac{-1}{4}\right)^4$

**Solution:**

(i) $(-5)^3 = (-5) \times (-5) \times (-5)$
$= (+25) \times (-5)$
$= -125$

Thus, $(-5)^3 = -125$

(ii) $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3}$

$= \frac{2\times 2}{3\times 3} = \frac{4}{9}$

Thus, $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(iii) $\left(\frac{-1}{4}\right)^4 = \left(-\frac{1}{4}\right)\times\left(-\frac{1}{4}\right)\times\left(-\frac{1}{4}\right)\times\left(-\frac{1}{4}\right)$

$= \frac{-1\times -1\times -1\times -1}{4\times 4\times 4\times 4} = \frac{1}{256}$

Thus, $\left(\frac{-1}{4}\right)^4 = \frac{1}{256}$

## EXERCISE 4.1

**1.** Identify the exponent and base in each of the following.

(i) $(-1)^9$ (ii) $2^{100}$ (iii) $(-19)^{22}$ (iv) $3^{-5}$

(v) $(ab)^n$ (vi) $\left(\frac{-6}{11}\right)^8$ (vii) $a^{-mn}$ (viii) $\left(\frac{2}{9}\right)^7$

(ix) $\left(\frac{p}{q}\right)^4$ (x) $\left(-\frac{1}{x}\right)^6$ (xi) $\left(\frac{x}{y}\right)^m$ (xii) $\left(\frac{11}{13}\right)^b$

**2.** Express each of the following in exponential form.

(i) $5 \times 5 \times 5 \times 5$ (ii) $\frac{-3}{7}\times\frac{-3}{7}\times\frac{-3}{7}\times\frac{-3}{7}$

(iii) $p \times p \times p \times p \times p$ (iv) $\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}$

(v) $xy \times xy \times xy$ (vi) $31 \times 31 \times 31 \times 31 \times 31$

(vii) $(-a) \times (-a) \times (-a) \times (-a) \times (-a) \times (-a) \times (-a)$





**3.** Prove that:

(i) $(5)^3 = 125$ (ii) $(-1)^{11} = -1$ (iii) $(-3)^5 = -243$

(iv) $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$ (v) $\left(-\frac{1}{8}\right)^3 = -\frac{1}{512}$ (vi) $\left(\frac{-2}{3}\right)^6 = \frac{64}{729}$

(vii) $\left(\frac{1}{10}\right)^4 = \frac{1}{10000}$ (viii) $\left(-\frac{4}{3}\right)^3 = \frac{-64}{27}$ (xi) $\left(\frac{2}{5}\right)^4 = \frac{16}{625}$

**4.** Express each rational number using an exponent.

(i) 121 (ii) 81 (iii) –625

(iv) $\frac{1}{1000}$ (v) $\frac{8}{343}$ (vi) $-\frac{1}{32}$

## 4.2 Laws of Exponents/Indices

Exponents are used in solving many problems, so it is important that we understand the laws for working with exponents. Let us discuss these laws one by one, and see some examples.

### 4.2.1 Using Rational Numbers to Deduce Laws of Exponents

- **Product Law**
- When bases are same but exponents are different

**Consider the following examples**

$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$
$= 2 \times 2 \times 2 \times 2 \times 2$
$= 2^5$

From the above, we can notice that the same result can be obtained by adding the exponents of two numbers.

$2^3 \times 2^2 = 2^{3+2} = 2^5$

Similarly,

$\left(\frac{-3}{4}\right)^2 \times \left(\frac{-3}{4}\right)^2 = \left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)\times\left(\frac{-3}{4}\right)$

$$=\left(\frac{-3}{4}\right)^7$$

Again use the short method to find the result.

$$\left(\frac{-3}{4}\right)^2\times\left(\frac{-3}{4}\right)^5=\left(\frac{-3}{4}\right)^{2+5}=\left(\frac{-3}{4}\right)^7$$

From the above examples, we can deduce the following law:

"While multiplying two rational numbers with the same base, we add their exponents but the base remains unchanged, i.e. for any number "*a*" with exponents *m* and *n*, this law is written as,

$$a^m \times a^n = a^{m+n}$$

- **When bases are different but exponents are same**

We know that

$$2^3 \times 5^3 = (2 \times 2 \times 2) \times (5 \times 5 \times 5)$$
$$= (2\times5) \times (2\times5) \times (2\times5)$$
$$= (2\times5)^3$$

Similarly,

$$\left(\frac{-1}{4}\right)^3\times\left(\frac{3}{4}\right)^3=\left[\left(\frac{-1}{4}\right)\times\left(\frac{-1}{4}\right)\times\left(\frac{-1}{4}\right)\right]\times\left[\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\right]$$
$$=\left(-\frac{1}{4}\times\frac{3}{4}\right)\times\left(-\frac{1}{4}\times\frac{3}{4}\right)\times\left(-\frac{1}{4}\times\frac{3}{4}\right)=\left(-\frac{1}{4}\times\frac{3}{4}\right)^3$$

From the above examples, we can deduce the following law:

"While multiplying two rational numbers having the same exponent, the product of the two bases is written with the given exponent." Suppose two rational numbers are "*a*" and "*b*" with exponent "*n*" then,

$$a^n \times b^n = (ab)^n$$

**Example:** Simplify the following expressions.

(i) $5^3 \times 5^4$ (ii) $(-3)^3 \times (-2)^3$

(iii) $\left(\frac{-1}{4}\right)^2\times\left(\frac{2}{3}\right)^2$ (iv) $\left(\frac{-3}{2}\right)^3\times\left(\frac{-3}{2}\right)^4$





**Solution:**

(i) $5^3 \times 5^4$

$= 5^{3+4} = 5^7$ [$\because a^m \times a^n = a^{m+n}$]

(ii) $(-3)^3 \times (-2)^3$

$= [(-3) \times (-2)]^3 = [6]^3$ [$\because a^n \times b^n = (ab)^n$]

(iii) $\left(\frac{-1}{4}\right)^2\times\left(\frac{2}{3}\right)^2$

$=\left[\left(\frac{-1}{4}\right)\times\left(\frac{2}{3}\right)\right]^2$ [$\because a^n \times b^n = (ab)^n$]

$=\left[\frac{-1\times2}{4\times3}\right]^2=\left[\frac{-1}{6}\right]^2$

(iv) $\left(\frac{-3}{2}\right)^3\times\left(\frac{-3}{2}\right)^4$ [$\because a^m \times a^n = a^{m+n}$]

$=\left(\frac{-3}{2}\right)^{3+4}=\left(\frac{-3}{2}\right)^7$

## EXERCISE 4.2

**1.** Simplify the using the laws of exponent into the exponential form.

(i) $(-4)^5 \times (-4)^6$ (ii) $m^3 \times m^4$ (iii) $\left(\frac{2}{7}\right)^3\times\left(\frac{2}{7}\right)^2$

(iv) $\left(\frac{1}{10}\right)^4\times\left(\frac{1}{10}\right)^5$ (v) $p^{10} \times q^{10}$ (vi) $\left(\frac{2}{5}\right)^3\times\left(\frac{5}{7}\right)^3$

(vii) $\left(\frac{-1}{2}\right)^6\times\left(\frac{-1}{2}\right)^5$ (viii) $(-3)^7 \times (-5)^7$ (ix) $\left(\frac{2}{3}\right)^{10}\times\left(\frac{2}{3}\right)^7$

(x) $\left(\frac{-10}{11}\right)^7\times\left(\frac{-10}{11}\right)^6$ (xi) $\left(\frac{11}{7}\right)^8\times\left(\frac{21}{22}\right)^8$

(xii) $\left(\frac{-x}{y}\right)\times\left(\frac{-x}{y}\right)^{11}$

**2.** Verify the following by using the laws of exponent.

(i) $(3 \times 5)^4 = 3^4 \times 5^4$ (ii) $(7 \times 9)^8 = 7^8 \times 9^8$

(iii) $(2)^6 \times (2)^3 = 2^9$ (iv) $(x \times y)^m = x^m y^m$

(iv) $(8)^5 \times (8)^7 = (8)^{12}$ (v) $(p)^r \times (p)^s = p^{r+s}$

- **Quotient Law**
- **When bases are same but exponents are different**

Consider the following.

$$\frac{2^7}{2^3} = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2}$$
$$= 2\times2\times2\times2 = 2^4$$

Let us find the same quotient by another way.

$$\frac{2^7}{2^3} = 2^{7-3} = 2^4$$

Similarly,

$$\left(\frac{-2}{3}\right)^5 \div \left(\frac{-2}{3}\right)^2 = \frac{\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)}{\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)}$$
$$= \left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right) = \left(\frac{-2}{3}\right)^3$$

According to the short method that we used for finding the quotient:

$$\left(-\frac{2}{3}\right)^5 \div \left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)^{5-2} = \left(-\frac{2}{3}\right)^3$$

**Thus, from the above examples we can suggest another law;**

"The division of two rational numbers with the same base can be performed by subtracting their exponents". Suppose 'a' is the base of any two rational numbers with exponents '$m$' and '$n$' such that $a \neq 0$ and $m > n$, then,

$$a^m \div a^n = a^{m-n}$$

- **When bases are different but exponents are same**

We know that: $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)$





$$= \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2^4}{3^4} = 2^4 \div 3^4$$

Similarly,

$$\left(\frac{x}{y}\right)^5 = \left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)$$
$$= \frac{x\times x\times x\times x\times x}{y\times y\times y\times y\times y} = \frac{x^5}{y^5} = x^5 \div y^5$$

Thus, this law can be written as:

For any two rational numbers "$a$" and "$b$", where $b \neq 0$ and "$n$" is their exponent, then,

$$a^n \div b^n = \left(\frac{a}{b}\right)^n$$

**Example:** Simplify.

(i) $9^8 \div 3^8$ (ii) $\left(-\frac{3}{11}\right)^7 \div \left(-\frac{3}{11}\right)^4$ (iii) $\left(\frac{3}{7}\right)^9 \div \left(\frac{3}{7}\right)^2$

(iv) $(14)^{11} \div (63)^{11}$

**Solution:**

(i) $9^8 \div 3^8$

$$= \left(\frac{9}{3}\right)^8 = 3^8 \quad \because a^n \div b^n = \left(\frac{a}{b}\right)^n$$

(ii) $\left(-\frac{3}{11}\right)^7 \div \left(-\frac{3}{11}\right)^4$

$$= \left(\frac{-3}{11}\right)^{7-4} = \left(-\frac{3}{11}\right)^3 \quad \because a^m \div a^n = a^{m-n}$$

(iii) $\left(\frac{3}{7}\right)^{9-2} \div \left(\frac{3}{7}\right)^2$

$$= \left(\frac{3}{7}\right)^{9-2} = \left(\frac{3}{7}\right)^7 \quad \because a^m \div a^n = a^{m-n}$$

(iv) $(14)^{11} \div (63)^{11}$

$$= \left(\frac{14}{63}\right)^{11} = \left(\frac{2}{9}\right)^{11} \quad \because a^n \div b^n = \left(\frac{a}{b}\right)^n$$

## EXERCISE 4.3

**1.** Simplify

| | | | | | |
|---|---|---|---|---|---|
| (i) | $2^7 \div 2^2$ | (ii) | $(-9)^{11} \div (-9)^8$ | (iii) | $(3)^4 \div (5)^4$ |
| (iv) | $(m)^3 \div (n)^3$ | (v) | $(a)^7 \div (a)^2$ | (vi) | $(b)^p \div (b)^q$ |
| (vii) | $\left(\frac{3}{4}\right)^7 \div \left(\frac{3}{4}\right)^2$ | (viii) | $\left(\frac{1}{6}\right)^{15} \div \left(\frac{1}{6}\right)^{11}$ | (ix) | $(2)^5 \div (3)^5$ |
| (x) | $\left(\frac{-3}{10}\right)^{17} \div \left(\frac{-3}{10}\right)^8$ | (xi) | $(x)^a \div (y)^a$ | (xii) | $\left(\frac{p}{q}\right)^{23} \div \left(\frac{p}{q}\right)$ |

**2.** Prove that

(i) $2^4 \div 7^4 = \left(\frac{2}{7}\right)^4$ (ii) $(-4)^3 \div (5)^3 = \left(\frac{-4}{5}\right)^3$ (iii) $3^8 \div 3 = 3^7$

(iv) $a^6 \div b^6 = \left(\frac{a}{b}\right)^6$ (v) $\left(\frac{-21}{22}\right)^7 \div \left(\frac{-21}{22}\right)^3 = \left(\frac{-21}{22}\right)^4$

(vi) $\left(\frac{-9}{13}\right)^5 \div \left(\frac{-9}{13}\right)^4 = \left(\frac{-9}{13}\right)$

- **Power Law**

We have studied that $a^m \times a^n = a^{m+n}$. Let us use this law to simplify an expression $(3^4)^2$.

$$(3^4)^2 = 3^4 \times 3^4$$
$$= 3^{4+4} = 3^8 \text{ is the same as } 3^{4\times2}$$

We solve another expression using the same law.

$$\left[\left(\frac{-1}{2}\right)^7\right]^2 = \left(\frac{-1}{4}\right)^7 \times \left(\frac{-1}{2}\right)^7$$
$$= \left(\frac{-1}{2}\right)^{7+7} = \left(\frac{-1}{2}\right)^{14} \text{ is also the same as } \left(\frac{-1}{2}\right)^{7\times2}$$

Thus, from the above examples, we can deduce that the base remains the same with a new exponent equal to the product of the two exponents, that is: $(a^m)^n = a^{m\times n} = a^{mn}$

- **Zero Exponent**

By the quotient law, we know that anything divided by itself is 1 as shown below.





$$\frac{3^2}{3^2} = \frac{3\times3}{3\times3} = 1$$

This can also be written as $3^{2-2} = 3^0 = 1$

Similarly,

$$\frac{(-2)^4}{(-2)^4} = \frac{(-2)\times(-2)\times(-2)\times(-2)}{(-2)\times(-2)\times(-2)\times(-2)} = 1$$

This can also be written as $(-2)^{4-4} = (-2)^0 = 1$.

Thus, we can define this law as:

Any non-zero rational number with zero exponent is equal to 1. Suppose "$a$" be any non-zero rational number with exponent "0", then $a^0 = 1$

- **Negative Exponents**

Look at the pattern given below.

$10^2 = 10 \times 10$

$10^1 = 10$

$10^0 = 1$

$$10^{-1} = \frac{1}{10}$$

$$10^{-2} = \frac{1}{10}\times\frac{1}{10} = \frac{1}{10\times10} = \frac{1}{10^2}$$

..................................................

..................................................

..................................................

$$10^{-m} = \frac{1}{10\times10\times\cdots\times10(m \text{ times})} = \frac{1}{10^m}$$

In general, it can be written as; $a^{-m} = \frac{1}{a^m}$

We can also deduce this law from $a^m \times a^n = a^{m+n}$. Suppose $n = -m$, then we will get,

$$a^m \times a^{-m} = a^{m-m} \Rightarrow a^m \times a^{-m} = a^0 \Rightarrow a^m \times a^{-m} = 1 \because a^0 = 1$$

Divided by $a^m$ on both sides.

$$\frac{a^m \times a^{-m}}{a^m} = \frac{1}{a^m} \Rightarrow a^{-m} = \frac{1}{a^m}$$

Thus, we have another law:

Any non-zero number raised to any negative power is equal to its reciprocal raised to the opposite positive power. i.e.

$$a^{-m} = \frac{1}{a^m}$$

If $\frac{p}{q}$ is a non-zero rational number, then according to the above given law, we have: $\left(\frac{p}{q}\right)^{-m} = \frac{1}{\left(\frac{p}{q}\right)^m} = \frac{1}{\frac{p^m}{q^m}} = \frac{p^m}{q^m} = \left(\frac{p}{q}\right)^m$

Thus, $\left(\frac{p}{q}\right)^{-m} = \left(\frac{p}{q}\right)^m$

**Example 1:** Express the following as a single exponent.

(i) $(3^4)^5$ (ii) $\left[\left(\frac{-2}{3}\right)^3\right]^2$ (iii) $\left[\left(\frac{1}{7}\right)^5\right]^6$

**Solution:**

(i) $(3^4)^5$ $\because (a^m)^n = a^{mn}$

$= 3^{4\times5}$

$= 3^{20}$

(ii) $\left[\left(\frac{-2}{3}\right)^3\right]^2$ $\because (a^m)^n = a^{mn}$

$= \left(\frac{-2}{3}\right)^{3\times2} = \left(\frac{-2}{3}\right)^6$

(iii) $\left[\left(\frac{1}{7}\right)^5\right]^6$ $\because (a^m)^n = a^{mn}$

$= \left(\frac{1}{7}\right)^{5\times6} = \left(\frac{1}{7}\right)^{30}$

**Example 2:** Change the following negative exponents into positive exponents.

(i) $\left(\frac{3}{4}\right)^{-3}$ (ii) $\left(\frac{-2}{5}\right)^{-4}$ (iii) $\left(\frac{a}{-b}\right)^{-6}$

**Solution:** (i) $\left(\frac{3}{4}\right)^{-3}$

$= \frac{1}{\left(\frac{3}{4}\right)^3}$ $\because a^{-m} = \frac{1}{a^m}$

$= \frac{1}{\frac{3^3}{4^3}} = \frac{4^3}{3^3} = \left(\frac{4}{3}\right)^3$

Thus, $\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3$





(ii) $\left(\frac{-2}{5}\right)^{-4}$

$= \frac{1}{\left(\frac{-2}{5}\right)^4}$ $\because a^{-m} = \frac{1}{a^m}$

$= \frac{1}{\frac{(-2)^4}{5^4}} = \frac{5^4}{(-2)^4} = \left(\frac{5}{-2}\right)^4$ or $\left(\frac{-5}{2}\right)^4$

Thus, $\left(\frac{-2}{5}\right)^{-4} = \left(\frac{-5}{2}\right)^4$

(iii) $\left(\frac{a}{-b}\right)^{-6}$

$= \frac{1}{\left(\frac{a}{-b}\right)^6}$ $\because a^{-m} = \frac{1}{a^m}$

$= \frac{1}{\frac{a^6}{(-b)^6}} = \frac{(-b)^6}{a^6} = \left(\frac{-b}{a}\right)^6$

Thus, $\left(\frac{a}{-b}\right)^{-6} = \left(\frac{-b}{a}\right)^6$

## 4.2.2 Demonstration of the concept of Power of an Integer

We know that when we multiply a negative number by itself, it gives a positive result because minus time minus is plus. For example,

$(-3) \times (-3) = (-3)^2 = +9$ $(-5) \times (-5) = (-5)^2 = +25$

But do you know it happens to all even exponents that can be seen in the pattern given below.

$(-2)^2 = (-2) \times (-2) = +4$ ............ (*even*)
$(-2)^3 = (-2) \times (-2) \times (-2) = -8$ ............ (*odd*)
$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = +16$ ............ (*even*)
$(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$ ............ (*odd*)
$(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = +64$ ............ (*even*)

From the above it can also be noticed that a negative number with an odd exponent gives a negative result. So, we can explain it as:

Let "*a*" be any positive rational number and "n" be any non-zero integer, than according to this law:

- If "*n*" is an even integer, then $(-a)^n$ is positive.
- If "*n*" is an odd integer, then $(-a)^n$ is negative.

### 4.2.3 Applying Laws of Exponent to Evaluate Expressions

**Example 3:** Simplify and express the result in the simple form.

(i) $(4^7 \div 4^5) \times 2^2$ (ii) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{3} \times \left(\frac{3}{5}\right)^{5} \times \left(\frac{3}{5}\right)^{-5}$

(iii) $\left(\frac{-2}{7}\right)^{5} \times \left(\frac{-2}{7}\right)^{-2} \times \left[\left(\frac{-2}{7}\right)^{2}\right]^{-1}$

**Solution:**

(i) $(4^7 \div 4^5) \times 2^2$

$= 4^{7-5} \times 2^2$

$= 4^2 \times 2^2$ $\quad \because a^m \div a^n = a^{m-n}$

$= (4 \times 2)^2$ $\quad \because a^n \times b^n = (ab)^n$

$= 8^2 = 64$

(ii) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{3} \times \left(\frac{3}{5}\right)^{5} \times \left(\frac{3}{5}\right)^{-5}$

$\left(\frac{2}{5}\right)^{-3+3} + \left(\frac{3}{5}\right)^{5+(-5)}$ $\quad \because a^m \times a^n = a^{m+n}$

$\left(\frac{2}{5}\right)^{0} + \left(\frac{3}{5}\right)^{0}$

$1 + 1 = 2$ $\quad \because a^0 = 1$

(iii) $\left(\frac{-2}{7}\right)^{5} \times \left(\frac{-2}{7}\right)^{-2} \times \left[\left(\frac{-2}{7}\right)^{2}\right]^{-1}$

$= \left(\frac{-2}{7}\right)^{5+(2)} \times \left(\frac{-2}{7}\right)^{2\times(-1)}$ $\quad \because a^m \times a^n = a^{m+n}$, $(a^m)^n = a^{mn}$

$= \left(\frac{-2}{7}\right)^{3} \times \left(\frac{-2}{7}\right)^{-2}$

$= \left(\frac{-2}{7}\right)^{3+(-2)}$ $\quad \because a^m \times a^n = a^{m+n}$

$= \left(\frac{-2}{7}\right)^{3-2} = \frac{-2}{7}$





## EXERCISE 4.4

1. Express the following as single exponents.

(i) $(2^3)^5$ (ii) $(10^2)^2$ (iii) $(-3^4)^5$

(iv) $(p^2)^3$ (v) $(-m^7)^4$ (vi) $(x^a)^b$

(vii) $\left[\left(\frac{-1}{3}\right)^{3}\right]^{3}$ (viii) $\left[\left(\frac{2}{9}\right)^{3}\right]^{6}$ (ix) $\left[\left(\frac{p}{q}\right)^{m}\right]^{n}$

2. Change the following negative exponents into positive exponents.

(i) $(12)^{-3}$ (ii) $(-a)^{-2}$ (iii) $(100)^{-5}$

(iv) $\left(\frac{2}{3}\right)^{-4}$ (v) $\left(\frac{-1}{10}\right)^{-9}$ (vi) $\left(\frac{x}{y}\right)^{-b}$

3. Evaluate the following expressions.

(i) $(1^2)^3 \times (2^3)^2$ (ii) $[(-3)^7]^0 \times [(-3)^2]^2$

(iii) $\left[\left(\frac{-3}{4}\right)^{0}\right]^{3} \times \left[\left(\frac{-3}{4}\right)^{2}\right]^{2}$ (iv) $\left(\frac{2^3}{2^6 \div 2^3}\right)$

(v) $\dfrac{\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-6}}{\left(\frac{1}{2}\right)^{-5}}$ (vi) $\dfrac{\left(\frac{-2}{9}\right)^{5} \times \left(\frac{-2}{9}\right)^{-5}}{\left(\frac{3}{2}\right)^{4} \times \left(\frac{3}{2}\right)^{-4}}$

(vii) $\dfrac{\left(\frac{1}{3}\right)^{-3}}{\left(\frac{1}{3}\right)^{-4}} - \dfrac{\left(\frac{1}{3}\right)^{-5}}{\left(\frac{1}{3}\right)^{-6}}$ (viii) $\dfrac{\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{4}}{\left(\frac{2}{3}\right)^{-4} \times \left(\frac{2}{3}\right)^{-4}}$

(ix) $\left(\frac{2}{3}\right)^{3} \times \left(\frac{2}{3}\right)^{0} \times \left(\frac{2}{3}\right)^{-3}$ (x) $\left(\frac{-1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-4}$

## REVIEW EXERCISE 4

1. Answer the following questions.
   (i) What is meant by the exponent of a number?
   (ii) What is the product law with the same base?
   (iii) Define the power law of exponent.
   (iv) What is the reciprocal of $\frac{p}{q}$ ?

2. Fill in the blanks.
   (i) 5×5×5×5 can be written in exponential form as _________.
   (ii) $a^n \times b^n$ = __________ .
   (iii) $a^n \div b^n$ = ___________.
   (iv) Any non-zero rational number with _________ exponent equals to 1.
   (v) $(-a)^n$ is positive, if '*n*' is an _______ integer.
   (vi) _________ is read as '*n*th power of *a*'.

3. Tick (✓) the correct answer.

4. Find the value of:
   (i) $(4)^{-3}$  (ii) $(-5)^4$  (iii) $(2)^{-9}$
   (iv) $\left(\frac{-1}{3}\right)^{-5}$  (v) $\left(\frac{3}{10}\right)^{3}$  (vi) $-\left(\frac{11}{13}\right)^{2}$





5. Use the laws of exponents to find the value of $x$.
   (i) $[(-7)^3]^6 = 7^x$  (ii) $\left[\left(\frac{3}{4}\right)^2\right]^5 = \frac{3^x}{4^x}$
   (iii) $\left[\left(\frac{13}{8}\right)^4\right]4 = \frac{13^x}{8^x}$  (iv) $\left(\frac{5}{3}\right)^5 \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$
   (v) $\left(\frac{2}{9}\right)^2 \div \left(\frac{2}{9}\right)^9 = \left(\frac{2}{9}\right)^{2x-1}$

6. Simplify and write the answer in simple form.
   (i) $\left[\left(\frac{-3}{4}\right)^2 \times \left(\frac{-3}{4}\right)^3\right] \div \left[\left(\frac{-3}{4}\right)^2\right]^2$
   (ii) $\left(\frac{5}{19}\right)^{10} \times \left[\left(\frac{5}{19}\right)^2\right]^3 \div \left[\left(\frac{5}{19}\right)^4\right]^4$
   (iii) $\left[\left(\frac{18}{11}\right)^3 \div \left(\frac{18}{11}\right)^2\right]^5 \div \left[\left(\frac{18}{11}\right)^2\right]^2$
   (iv) $\left[\left(\frac{-4}{9}\right)^2\right]^8 \div \left[\left(\frac{-4}{9}\right)^3\right]^5 \times \left(\frac{-4}{9}\right)$
   (v) $\left[\left(\frac{1}{10}\right)^3\right]^2 \times \left[\left(\frac{1}{10}\right)^6\right]^3 \div \left(\frac{1}{10}\right)^{25}$

## SUMMARY

- The exponent of a number indicates us how many times a number (base) is multiplied with itself.
- While multiplying two rational numbers with the same base, we add their exponents but the base remains unchanged. i.e. $a^m \times a^n = a^{m+n}$
- While multiplying two rational numbers having same exponent, the product of two bases is written with the given exponent. i.e. $a^n \times b^n = (ab)^n$

- The division of two rational numbers with the same base can be performed by subtracting their exponents. i.e. $a^m \div a^n = a^{m-n}$
- To raise a power to another power, we just write the product of two exponents with the same base. i.e. $(a^m)^n = a^{mn}$
- Any non-zero rational number with zero exponent equals to 1, i.e. $a^0 = 1$
- Any non-zero rational number with a negative exponent equals to its reciprocal with the same but positive exponent. i.e. $a^{-m} = \frac{1}{a^m}$
- $(-a)^n$ is positive, if $n$ is an even integer and $(-a)^n$ is negative, if $n$ is an odd integer.

CHAPTER

5

# SQUARE ROOT OF POSITIVE NUMBER

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Define a perfect square.
- Test whether a number is a perfect square or not.
- Identify and apply the following properties of perfect square of a number.
  - The square of an even number is even.
  - The square of an odd number is odd.
  - The square of a proper fraction is less than itself.
  - The square of a decimal less than 1 is smaller than the decimal.
- Define the square root of a natural number and recognize its notation.
- Find square root, by division method and factorization method of a
  - Natural number,
  - Fraction,
  - Decimal,

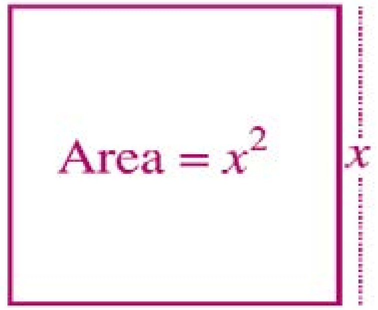
  Which are perfect squares.
- Solve real life problems involving square roots.

# 5.1 Introduction

In previous classes, we have learnt that the area of a square can be calculated by multiplying its length by itself as shown below.

$$\text{Area of the square} = \text{length} \times \text{length}$$
$$= x \times x$$
$$= x^2$$

It means $x^2$ is an area of a square whose side length is $x$ or simply we can say that "$x^2$ is the square of $x$". i.e.
The square of $x = x^2$







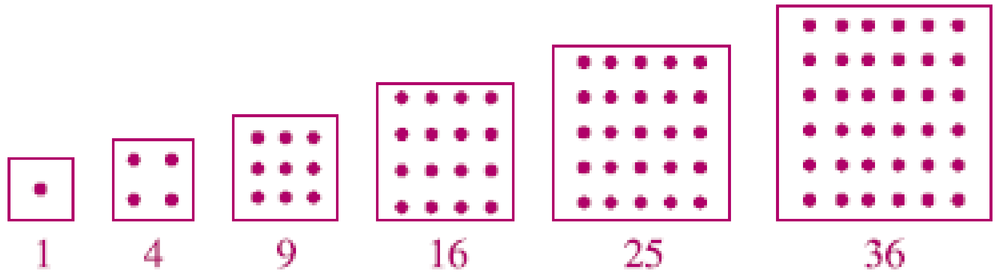

Thus, the square of a number can be defined as:
"The product of a number with itself is called its square."

### 5.1.1 Perfect Squares

A natural number is called a perfect square, if it is the square of any natural number. To make it clear, let us find the squares of some natural numbers.

| | |
|---|---|
| $1^2 = 1 \times 1 = 1$ | $6^2 = 6 \times 6 = 36$ |
| $2^2 = 2 \times 2 = 4$ | $7^2 = 7 \times 7 = 49$ |
| $3^2 = 3 \times 3 = 9$ | $8^2 = 8 \times 8 = 64$ |
| $4^2 = 4 \times 4 = 16$ | $9^2 = 9 \times 9 = 81$ |
| $5^2 = 5 \times 5 = 25$ | $10^2 = 10 \times 10 = 100$ and so on |

Here, "1 is the square of 1", "4 is the square of 2", "9 is the square of 3" and so on. It can be noticed that all these are natural numbers. So, these are perfect squares which can be represented by drawing dots in squares.




When we have a number of rows equal to number of dots in a row, then it shows a perfect square.

### 5.1.2 To Test whether a number is a Perfect Square or not

To check whether a given number is a perfect square or not, write the number as a product of its prime factors, if all the factors can be grouped in pairs, then the given number is a perfect square.

**Example 1:** Check whether the following numbers are perfect squares or not.

(i) 3969 (ii) 6084 (iii) 3872

**Solution:**

(i) 3969

The prime factors of 3969 = $\lfloor 3\times 3\rfloor \times \lfloor 3\times 3\rfloor \times \lfloor 7\times 7\rfloor$

We can see that each factor forms a pair. Hence, 3969 is a perfect square.

| | |
|---|---|
| 3 | 3969 |
| 3 | 1323 |
| 3 | 441 |
| 3 | 147 |
| 7 | 49 |
| | 7 |

(ii) 6084

The prime factors of 6084 = $\lfloor 2\times 2\rfloor \times \lfloor 3\times 3\rfloor \times \lfloor 13\times 13\rfloor$

Here, each factor of 6084 forms a pair. So, it is a perfect square.

| | |
|---|---|
| 2 | 6084 |
| 2 | 3042 |
| 3 | 1521 |
| 3 | 507 |
| 13 | 169 |
| | 13 |

(iii) 3872

The prime factors of 3872 = $\lfloor 2\times 2\rfloor \times \lfloor 2\times 2\rfloor \times 2 \times \lfloor 11\times 11\rfloor$

We can see that 2 is a factor which cannot be paired with any equal factor. So, 3872 is not a perfect square.

| | |
|---|---|
| 2 | 3872 |
| 2 | 1936 |
| 2 | 968 |
| 2 | 484 |
| 2 | 242 |
| 11 | 121 |
| | 11 |

## 5.1.3 Properties of Perfect Squares of Numbers

There are some interesting properties about perfect squares. Let us discuss some of them.

- The square of an even number is even

We know that natural numbers can be divided into two groups: even numbers and odd numbers. Look at the squares of the even numbers given below.

$2^2 = 2 \times 2 = 4$ $\quad 4^2 = 4 \times 4 = 16$

$6^2 = 6 \times 6 = 36$ $\quad 8^2 = 8 \times 8 = 64$

$10^2 = 10 \times 10 = 100$ $\quad 12^2 = 12 \times 12 = 144$





Notice that the squares of all even numbers are even numbers.

- **The square of an odd number is odd**

Now we find the square of some odd numbers.

$1^2 = 1 \times 1 = 1$ $\quad 3^2 = 3 \times 3 = 9$

$5^2 = 5 \times 5 = 25$ $\quad 7^2 = 7 \times 7 = 49$

$9^2 = 9 \times 9 = 81$ $\quad 11^2 = 11 \times 11 = 121$

Hence, the squares of all odd numbers are also odd numbers.

**Example 2:** Without solving, separate the perfect squares of even numbers and odd numbers

(i) 3481 (ii) 2704 (iii) 49284 (iv) 12321

**Solution:**

(i) 3481

The square of an odd number is also odd.

$\because$ 3481 is the square of an odd number.

(ii) 2704

The square of an even number is also even.

$\because$ 2704 is the square of an even number.

(iii) 49284

The square of an even number is also even.

$\because$ 49284 is the square of an even number.

(iv) 12321

The square of an odd number is also odd.

$\because$ 12321 is the square of an odd number.

- **The square of a proper fraction is less than itself**

To square a fraction, we multiply the numerator by itself and do the same for the denominator.

$$\left(\frac{2}{5}\right)^2 = \frac{2}{5}\times\frac{2}{5} = \frac{2\times 2}{5\times 5} = \frac{4}{25}$$

Now let us compare the fraction $\frac{2}{5}$ with its square $\frac{4}{25}$ by using the method of cross multiplication.

$$\frac{2}{5} \times \frac{4}{25} \qquad \boxed{50 > 20}$$

From the above it can be observed that the square of a proper fraction is less than itself ,i.e. $\frac{2}{5} > \frac{4}{25}$. Similarly,

$$\left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9} \qquad \boxed{\frac{1}{3} > \frac{1}{9}}$$

$$\left(\frac{4}{7}\right)^2 = \frac{4}{7} \times \frac{4}{7} = \frac{4 \times 4}{7 \times 7} = \frac{16}{49} \qquad \boxed{\frac{4}{7} > \frac{16}{49}}$$

- **The square of a decimal less than 1 is smaller than the decimal**

To find the square of a decimal, we can use the following method.

$$(0.3)^2 = (0.3) \times (0.3) = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100} = 0.09$$

Is 0.09 smaller than 0.3 or greater? Certainly, 0.09 is smaller than 0.3.i.e.0.09 < 0.3,

$$(0.02)^2 = (0.02) \times (0.02) = \frac{2}{100} \times \frac{2}{100} = \frac{4}{10000} = 0.0004$$

Again 0.0004 is smaller than 0.02 i.e. 0.0004 < 0.02.

It means the square of a decimal less than ‘1’ is always smaller than the given decimal.

## EXERCISE 5.1

**1.** Find the squares of the following numbers.

(i) 6 (ii) 5 (iii) 10 (iv) 7
(v) 13 (vi) 8 (vi) 41 (vii) 19
(ix) 100 (x) 9 (xi) 11 (xii) 25

**2.** Test whether the following numbers are perfect squares or not.

(i) 59 (ii) 625 (iii) 225 (iv) 196
(v) 425 (vi) 81 (vi) 121 (vii) 2500

**3.** Without solving, separate the perfect squares of even and odd numbers.

(i) 441 (ii) 144 (iii) 2401 (iv) 6561
(v) 2025 (vi) 11236 (vi) 7569 (vii) 12544





**4.** Find the squares of proper fractions. Also compare them with itself.

(i) $\frac{3}{4}$ (ii) $\frac{5}{6}$ (iii) $\frac{4}{11}$ (iv) $\frac{1}{7}$

**5.** Find the squares of decimals and compare them with itself.

(i) 0.4 (ii) 0.6 (iii) 0.12 (iv) 0.05

## 5.2 Square Roots

### 5.2.1 Defining square root of a natural number and recognizing its notation

The process of finding the square root is an opposite operation of “squaring a number”. To understand it, again we find some perfect squares.

$2^2$ = 4(2 squared is 4)
$5^2$ = 25( 5 squared is 25)
$7^2$ = 49(7 squared is 49)

These equations can also be read as, “2 is the square root of 4”, “5 is the square root of 25” and “7 is the square root of 49”.

Similarly, we can find the square root of any square number. For this purpose, we use the symbol “ $\sqrt{\ }$ ” to represent a square root, i.e. $\sqrt{x^2} = x$ where “ $\sqrt{\ }$ ” is called redical sign. Here, $x^2$ is called redicand.

If $x$ is any number that can be written in the form of $x = y^2$, then $x$ is called the square of y and y itself is called the square root of $x$.

### 5.2.2 Finding square roots by prime factorization

We have learnt that:

| If $a$, $b$ be any two numbers, then |
|---|
| (i) $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ |
| (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ |

The square root of 4 is, $\sqrt{4} = \sqrt{2^2} = 2$

The square root of 9 is, $\sqrt{9} = \sqrt{3^2} = 2$

The square root of 25 is, $\sqrt{25} = \sqrt{5^2} = 2$

and so on. But in case of large perfect squares, it becomes more difficult for us to guess their square roots. To solve this problem, we use a method which is called the prime factorization method.The steps for finding the method are given below,

**Step 1:** Find the prime factors of the given number. Suppose the given number is 36, then.

36 = 2 x 2 x 3 x3

| | |
|---|---|
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| | 3 |

**Step 2:** Take the square root on both sides.

$$\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3}$$

**Step 3:** Write them as a pair of prime factors of a perfect square.

$$\sqrt{36} = \sqrt{2 \times 2} \times \sqrt{3 \times 3}$$
$$= \sqrt{2^2} \times \sqrt{3^2}$$

**Step 4:** Write the square root of each perfect square, i.e. $\sqrt{x^2} = x$ and find their product.

$$\sqrt{36} = 2 \times 3 = 6$$

Hence, 6 is the square root of the given number 36.

| The prime factors of a perfect square are always in the pairs. |
|---|

**Example 1:** Write the square root of 900.

| | |
|---|---|
| 2 | 900 |
| 2 | 450 |
| 3 | 225 |
| 3 | 75 |
| 5 | 25 |
| | 5 |

**Solution:**

- Find the prime factors of 900.
  Factorization of 900 = 2 × 2 × 3 × 3 × 5 × 5





- Take square root on both sides.

$$\sqrt{900} = \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5}$$

Write them as a pair of prime factors of a perfect square.

$$\sqrt{900} = \sqrt{2 \times 2} \times \sqrt{3 \times 3} \times \sqrt{5 \times 5}$$
$$= \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{5^2}$$

Write the square root of each perfect square, i.e. $\sqrt{x^2} = x$ and find their product.

$$\sqrt{900} = 2 \times 3 \times 5 = 30$$

Hence, 30 is the square root of 900.

- **Finding Square Roots of Fractions**

We know that there are three types of common fractions.

- Proper fraction
- Improper fraction
- Compound fraction

**Example 2:** Find the square root of a common fraction $\frac{144}{256}$

**Solution:**

- We have to find the square root of $\frac{144}{256}$. So,

we can write it as: $\frac{\sqrt{144}}{\sqrt{256}} = \frac{\sqrt{144}}{\sqrt{256}}$

- Find separately the prime factors of 144 and 256 as given.

| | |
|---|---|
| 2 | 144 |
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| | 3 |

| | |
|---|---|
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| | 2 |

$$\frac{\sqrt{144}}{\sqrt{256}}=\frac{\sqrt{2\times2\times2\times2\times3\times3}}{\sqrt{2\times2\times2\times2\times2\times2\times2\times2}}=\frac{\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{3\times3}}{\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2\times2}}$$
$$=\frac{\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{3^2}}{\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}}=\frac{2\times2\times3}{2\times2\times2\times2}=\frac{12}{16}$$

Therefore, $\frac{12}{16}$ is the required answer.

**Example 3:** Find the square root of the compound fraction $1\frac{63}{81}$

**Solution:**

(i) Change the mixed fraction into an improper fraction as:

$$1\frac{63}{81}=\frac{144}{81}$$

Now find the square root. Thus,

| | |
|---|---|
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| | 3 |

| | |
|---|---|
| 2 | 144 |
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| | 3 |

$$\frac{\sqrt{144}}{\sqrt{81}}=\frac{\sqrt{2\times2\times2\times2\times3\times3}}{\sqrt{3\times3\times3\times3}}=\frac{\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{3\times3}}{\sqrt{3\times3}\times\sqrt{3\times3}}$$
$$=\frac{\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{3^2}}{\sqrt{3^2}\times\sqrt{3^2}}=\frac{2\times2\times3}{3\times3}=\frac{12}{9}=1\frac{3}{9}$$

Thus, $1\frac{3}{9}$ is the square root of $1\frac{63}{81}$

- **Finding Square Roots of Decimals**

In the case of decimals first we change them into common fractions and then we find the square root. After finding the square root, we again write the answer in decimal form. We make it clear with an example.





**Example 4:** Find the square root of the decimal 0.64

**Solution:**

- Change the decimal into a fraction as, $0.64=\frac{64}{100}$
- Now find the square root as a proper fraction.

| | |
|---|---|
| 2 | 64 |
| 2 | 32 |
| 2 | 26 |
| 2 | 8 |
| 2 | 4 |
| | 2 |

| | |
|---|---|
| 2 | 100 |
| 2 | 50 |
| 5 | 25 |
| | 5 |

$$\frac{\sqrt{64}}{\sqrt{100}}=\frac{\sqrt{2\times2\times2\times2\times2\times2}}{\sqrt{2\times2\times5\times5}}=\frac{\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2\times2}}{\sqrt{2\times2}\times\sqrt{5\times5}}$$
$$=\frac{\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}}{\sqrt{2^2}\times\sqrt{5^2}}=\frac{2\times2\times2}{2\times5}=\frac{8}{10}=0.8$$

Thus, 0.8 is the required square root of 0.64

## EXERCISE 5.2

**1.** Find the square roots of the following numbers.

(i) 4 (ii) $(9)^2$ (iii) 36 (iv) $(25)^2$
(v) 16 (vi) $c^2$ (vii) 49 (viii) $a^2$
(ix) 25 (x) 81 (xi) $y^2$ (xii) 100

**2.** Find the square roots of the following numbers by prime factorization.

(i) 144 (ii) 256 (iii) 576 (iv) 324
(v) 441 (vi) 729 (vii) 196 (viii) 1225
(ix) 10000 (x) 1764 (xi) 4356

**3.** Find the square roots of the following fractions.

(i) $\frac{49}{81}$ (ii) 2.25 (iii) $\frac{144}{196}$ (iv) 0.0196
(v) $\frac{784}{441}$ (vi) $1\frac{13}{36}$ (vii) 3.24 (viii) 12.25

(ix) $3\frac{325}{900}$ (x) 59.29 (xi) $1\frac{252}{324}$ (xii) 1.5626

**4.** Prove each of the following by prime factorization.

(i) $\sqrt{9\times 36}=\sqrt{9}\times\sqrt{36}$ (ii) $\sqrt{144\times 4}=\sqrt{144}\times\sqrt{4}$

(iii) $\sqrt{64\times 25}=\sqrt{64}\times\sqrt{25}$ (iv) $\sqrt{81\times 100}=\sqrt{81}\times\sqrt{100}$

(v) $\sqrt{\frac{144}{9}}=\frac{\sqrt{144}}{\sqrt{9}}$ (vi) $\sqrt{\frac{256}{4}}=\frac{\sqrt{256}}{\sqrt{4}}$

(vii) $\sqrt{\frac{484}{121}}=\frac{\sqrt{484}}{\sqrt{121}}$ (viii) $\sqrt{\frac{576}{144}}=\frac{\sqrt{576}}{\sqrt{144}}$

- **Finding Square Root by Division Method**

We have already learnt the process of finding the square root of natural numbers by prime factorization method. Now we learn another method for finding the square roots of natural numbers which is known as 'division method'.

**Example 1:** Find the square root of 324 by division method.

**Solution:** 324

**Step 1:** $\overline{3}\,\overline{24}$ From right to the left make pairs of the digits and show them by putting a bar over each of them.

**Step 2:** $1\,|\,\overline{3}\,\overline{24}$ Try to guess the greatest number whose square must be equal to or less than the first pair or digit (from left to right). Here we can see the required greatest number is 1.

**Step 3:**

```
      1
 1 | 3 24
   - 1
     2 24
```

Subtract the square of the number from the pair or digit. i.e. $1^2 = 1$ and $3 - 1 = 2$. Now bring down the 2nd pair as shown.

**Step 4:**

```
        1
  1  | 3 24
 +1  | -1
  2  |  2 24
```

Double the quotient and use it as 2nd divisor.





**Step 5:**

```
          18
  1   |  3 24
 +1   | -1
  2(8)|  2 24
      | -2 24
           0
```

Again try to guess the greatest number whose product with divisor must be equal to or less than the 2nd dividend as given in the opposite.

$25\times 5=125$
$26\times 6=156$
$27\times 7=189$
$28\times 8=224$

The quotient is the required square root. It can be checked by finding its square. Thus the required square root is 18.

**Example 2:** Find the square root of 585225 by division method.

**Solution:** 585225

```
            7 6 5
   7     | 58 52 25
  +7     | -49
  14(6)  |   9 52
   +6    |  -8 76
  152(5) |     76 25
         |    -76 25
                  0
```

∵ $6\times 6=36$
$7\times 7=49$
$8\times 8=64$

∵ $145\times 5=752$
$146\times 6=876$
$147\times 7=1029$

∵ $1524\times 4=6096$
$1525\times 5=7625$
$1526\times 6=9156$

Thus, the required square root is 765.

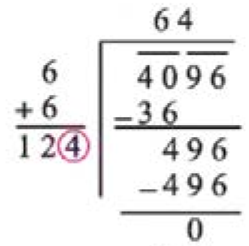
- **Finding Square Roots of Fractions**

We have learnt the method of finding the square root of fractions by prime factorization. Now we find the square root of a fraction by division method.

**Example 1:** Find the square roots of $\frac{4096}{15129}$ by division method.

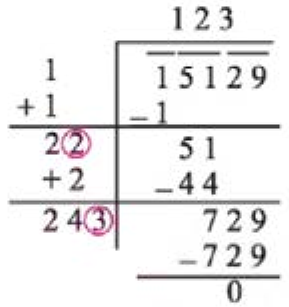
**Solution:** $\frac{4096}{15129}$

We know that: $\sqrt{\frac{4096}{15129}}=\frac{\sqrt{4096}}{\sqrt{15129}}$



∵ $5\times5=25$, $6\times6=36$, $7\times7=49$

∵ $123\times3=369$, $124\times4=496$



∵ $21\times1=21$, $22\times2=44$, $23\times3=69$

∵ $242\times2=484$, $243\times3=729$

Thus, $\sqrt{\frac{4096}{15129}}=\frac{\sqrt{4096}}{\sqrt{15129}}=\frac{64}{123}$

- **Finding Square Roots of Decimals**

To learn the process of finding the square roots of decimals, we examine the following example and its steps.

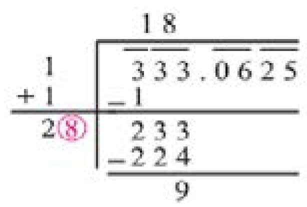
**Example 2:** Find the square root of 333.0625 by division method.

**Solution:** 333.0625

**Step 1:** Make the pairs of the whole number part of the decimal as number.*(from right to left)* $3\overline{33}.0625$

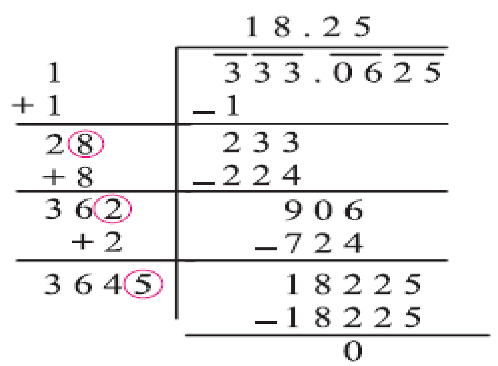
**Step 2:** Make the pairs of the decimal part. *(from left to right)* $\overline{3}\,\overline{33}\,.\,\overline{06}\,\overline{25}$

**Step 3:** Use the same division method as numbers.



∵ $27\times7=189$, $28\times8=224$, $29\times9=261$

**Step 4:** Put the decimal point in the quotient before bringing down the pair after decimal point.



∵ $361\times1=361$, $362\times2=724$, $363\times3=1089$

∵ $3644\times4=14576$, $3645\times5=18225$





Thus, $\sqrt{333.0625}=18.25$

**Example 3:** Find the square root of the following by division method.

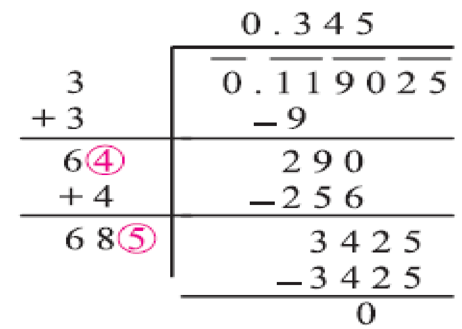
(i) 0.119025 (ii) 199.9396

**Solution:**

(i) 0.119025

Make pairs of the whole number part and decimal part respectively:

$0.\overline{11}\,\overline{90}\,\overline{25}$



∵ $2\times2=4$, $3\times3=9$, $4\times4=16$

∵ $63\times3=189$, $64\times4=256$, $65\times5=325$

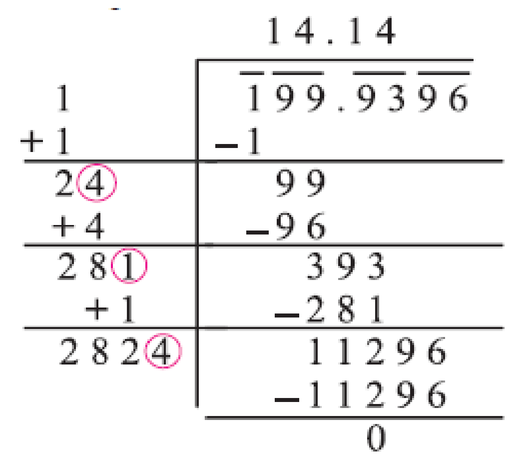
∵ $684\times4=2736$, $685\times5=3425$

Thus, $\sqrt{0.119025}=0.345$

(ii) 199.9396

Make pairs of the whole number part and decimal part respectively:

$\overline{1}\,\overline{99}\,.\,\overline{93}\,\overline{96}$



∵ $23\times3=69$, $24\times4=96$, $25\times5=125$

∵ $281\times1=281$, $282\times2=546$

∵ $2823\times3=8469$, $2824\times4=11296$

Thus, $\sqrt{199.9396}=14.14$

## EXERCISE 5.3

**1.** Find the square roots of the following by division method.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (i) | 729 | (ii) | 2304 | (iii) | 4489 | (iv) | 7056 |
| (v) | 9801 | (vi) | 14400 | (vii) | 15625 | (viii) | 18496 |
| (ix) | 207936 | (x) | 321489 | (xi) | 5499025 | (xii) | 4986289 |

**2.** Find the square roots of the following common fractions by division method.

(i) $\frac{36}{49}$ (ii) $\frac{225}{484}$ (iii) $\frac{81}{196}$ (iv) $\frac{729}{1024}$

(v) $2\frac{14}{25}$ (vi) $\frac{1296}{2025}$ (vii) $3\frac{526}{625}$ (viii) $\frac{3025}{4096}$

(ix) $2\frac{175}{225}$ (x) $\frac{324}{576}$ (xi) $\frac{5625}{40000}$ (xii) $1\frac{295}{729}$

**3.** Find the square roots of the following decimals by division method.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (i) | 0.0529 | (ii) | 1.5625 | (iii) | 9.7344 | (iv) | 0.4761 |
| (v) | 0.001369 | (vi) | 32.1489 | (vii) | 0.002025 | (viii) | 131.1025 |
| (ix) | 508.5025 | (x) | 799.7584 | (xi) | 1082.41 | (xii) | 4596.84 |

## 5.2.3 Solving Real Life Problems involving Square Root

We solve real life problems involving square roots by using the method of finding the square root.

**Example 1:** The area of a rectangular park is equal to another square shaped park. Find the length of a square shaped park if the length and breadth of the rectangular park are 81m and 25m respectively.

**Solution:**

Area of the rectangular park = length x breadth

= 81m x 25m = 2025 $m^2$

As we know that,

Area of square shaped park = Area of rectangular park

Length of side = $\sqrt{2025m}$

= $\sqrt{2025}$

= $\sqrt{3\times3\times3\times3\times5\times5}$

= $\sqrt{3^2\times3^2\times5^2}$

| | |
|---|---|
| 3 | 2025 |
| 3 | 675 |
| 3 | 225 |
| 3 | 75 |
| 5 | 25 |
| | 5 |





= $(3 \times 3 \times 5)$m = 45m

Thus, the required length is 45m.

**Example 2:** Find the length of a boundary of a square field whose area is 784$m^2$.

**Solution:**

Area of the square park = 784$m^2$

Length of side = $\sqrt{784}$

= $\sqrt{2\times2\times2\times2\times7\times7}$

= $\sqrt{2^2\times2^2\times7^2}$

= (2 x 2 x7)m = 28m

| | |
|---|---|
| 2 | 784 |
| 2 | 392 |
| 2 | 196 |
| 2 | 98 |
| 7 | 49 |
| | 7 |

The length of the boundary or perimeter of the square field:

= 4 (length)

= 4 (28m) = 112m

**Example 3:** Find the perimeter of a rectangular park whose length is three times of its width and the area is 720.75$m^2$. Also calculate the cost of fencing the park at the rate of Rs.195/m. *(use division method for finding square root)*

**Solution:**

We have

Length of the park = 3(width of the park)

Area of the rectangular park = 720.75$m^2$

(i) perimeter =? (ii) Cost of fencing =?

We know that: Area of the rectangular park = Length x width

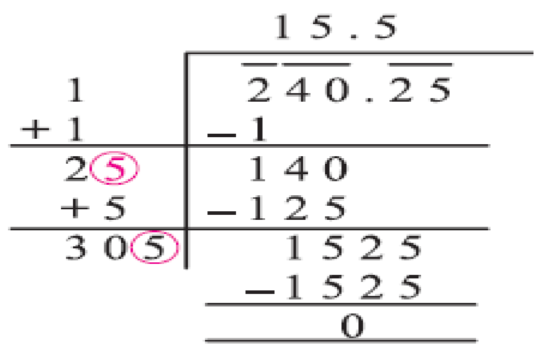
720.75$m^2$ = 3(width) x width

720.75$m^2$ = 3(width)$^2$

$\frac{720.75m^2}{3}$ = (width )$^2$

240.25$m^2$ = (width )$^2$

$\sqrt{240.25m^2}$ = width




∵ $24 \times 4 = 96$
$25 \times 5 = 125$
$26 \times 6 = 156$

∵ $304 \times 4 = 1216$
$305 \times 5 = 1525$

Thus, width $= \sqrt{240.25m^2} = 15.5m$

Length $= 3\ (\text{width}) = 3 \times 15.5m = 46.5m$

| | | |
|---|---|---|
| perimeter | = | 2(length + width) |
| | = | 2(46.5m + 15.5m)= 2 x 62m = 124m |
| Cost of fencing of 1m | = | Rs. 195 |
| Cost of fencing of 124m | = | Rs.(195 x 124) =24,185 |

## EXERCISE 5.4

1. The area of a square is $73.96m^2$. Calculate the length of its side.
2. 324 soldiers queued up such that the number of queues is equal to the number of soldiers in each queue. Find the number of queues.
3. By which smallest number can 275 be multiplied to get a perfect square?
4. By which smallest number can 648 be divided to get a perfect square?
5. The length and breadth of a rectangular swimming pool are 243m and 27m respectively. Find the length of a square shaped swimming pool which has the same area as rectangular swimming pool.
6. The base and height of a triangle are 8cm and 4.5cm respectively. Find the length of the side of a square whose area is the double of the given triangle.
7. The area of a square field is $617796m^2$. Find the length of its side.
8. A nursery owner tries to arrange 89500 plants into the shape of solid square. But he finds that he has 99 plants left over.





Find how many plants did the owner arrange in a row. (Hint = 89500 – 99= ?)

9. Which smallest number can be subtracted from 15198 to get a perfect square?
10. Find the number that gives 992.8801 after multiplying itself.
11. Find the measurement of the sides of a rectangle whose length is four times of its width and area is $51.84cm^2$.
12. The area of a circular swimming pool is $154m^2$. Find out the radius of the swimming pool.

## Review Exercise 5

**1.** Answer the following questions.

(i) What is meant by the square of a number?
(ii) Define a perfect square.
(iii) Which smallest number can be subtracted from 50 to get a perfect square?
(iv) Name the two methods for finding the square roots of large natural numbers.

**2.** Fill in the blanks.

(i) 4, 9, 16, 25, ... are called __________.
(ii) If $x = y^2$, then y is called the __________ of $x$.
(iii) While finding the square root by division method, the digits are paired from __________.
(iv) The number whose square root is non-terminiting and non-recurring decimal is called the __________ number.
(v) $\sqrt{\frac{121}{144}} = \frac{\sqrt{121}}{\square}$ (vi) $\sqrt{\frac{\square}{\square}} = \frac{\sqrt{169}}{\sqrt{100}}$

**3.** Tick( ✓ ) the correct answer.

**4.** Find the square root of the following.

(i) 1024 (ii) 484 (iii) $\frac{196}{49}$ (iv) 6.25

(v) 0.0225 (vi) $\frac{1225}{3025}$ (vii) $2\frac{14}{25}$ (viii) $1\frac{40}{81}$

(ix) 10.89 (x) $1\frac{23}{121}$ (xi) $\frac{225}{324}$ (xii) 3.0625

(xiii) 29.16 (xiv) $1\frac{539}{1225}$

**5.** Prove each of the following by prime factorization.

(i) $\sqrt{16\times 81}=\sqrt{16}\times\sqrt{81}$

(ii) $\sqrt{0.25\times 0.04}=\sqrt{0.25}\times\sqrt{0.04}$

(iii) $\sqrt{\frac{5625}{625}}=\frac{\sqrt{5625}}{\sqrt{625}}$

(iv) $\sqrt{\frac{5.76}{1.44}}=\frac{\sqrt{5.76}}{\sqrt{1.44}}$

**6.** 10201 soldiers have queued up for an attack such that the number of queues is equal to the number of the soldiers in each queue. Find the number of soldiers in each queue.

**7.** A businessman bought a square shaped park whose area is 50625m$^2$. He wants to fix light poles after the distance of each metre on its surroundings. For this he calculated the perimeter of the park. Do you know what perimeter he calculated?

**8.** The length and breadth of a rectangular swimming pool in a bungalow are 125m and 45m respectively. Find the length of another square shaped swimming pool which has the same area as rectangular swimming pool.





**9.** A teacher drew a triangle of 8cm height and 18cm base. Now he wants to draw a square whose area must be the twice that of the triangle. Calculate the length of the each side of the square that he has to draw.

**10.** Solve:

(i) By which smallest number can 605 be multiplied to get a perfect square?

(ii) By which smallest number can 3675 be divided to get a perfect square?

(iii) The area of a square is 94.09 m$^2$. What is the length of its side?

(iv) The length of a side of a square is 55.5 m. What is the area of the square?

## Summary

- The product of a number with itself is called its square.
- A natural number is called a perfect square, if it is a square of any natural number.
- The square of an even number is even and of an odd number is odd.
- The square of a proper fraction is less than itself.
- The square of a decimal less than 1 is smaller than itself.
- The process of finding the square root is the reverse operation of 'squaring a number'.
- If $x$ is a number such that $x = y^2$, then $x$ is known as the square of $y$ and $y$ is known as square root os $x$.
- To represent the square root, we use the symbol " $\sqrt{\phantom{x}}$ " which is called radical.
- To find the square root of a mixed fraction, we convert it into an improper fraction.
- We find the square root of a decimal by changing it into a fraction.
- We find the square root of a decimal by changing it into a fraction.

CHAPTER

6

# DIRECT AND INVERSE VARIATION

*Animation 6.1: Continued Ratio*
*Source & Credit: eLearn.Punjab*

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Define continued ratio and recall direct and inverse proportion.
- Solve real life problems (involving direct and inverse proportion) using unitary method and proportion method.
- Solve real life problems related to time and work using proportion.
- Find relation between time and distance (i.e. speed).
- Convert units of speed (kilometer per hour into meter per second and vice versa).
- Solve variation related problems involving time and distance.

# Introduction

Suppose someone uses 3 cups of water and 1 cup of milk to prepare tea. We can compare these two quantities by using the term of ratio. Water and milk are in the ratio of 3:1. Thus, the ratio is a comparison between the two or more quantities of the same kind which can be written by putting a colon (:) among them. A ratio is the numerical relation between two or more quantities having the same unit.

# 6.1 Continued Ratio

If the two ratios *a : b* and *b : c* are given for three quantities *a*, *b* and *c*, then the ratio *a : b : c* is called continued ratio which can be written as,

$$\begin{array}{ccccc} a & : & b & & \\ & & b & : & c \\ \hline a & : & b & : & c \end{array}$$

Here, ratio *a : b : c* is a continued ratio which is formed from the other two ratios *a* : *b* and *b* : *c* to express the relation between three quantities *a*, *b* and *c*.





From the above explanation, we can observe that b is a common element of two ratios which is the cause to combine them. Such type of an element is called the common member of the given ratios.

Always write the common member in the middle of the other elements according to the method given above.

**Example 1:** The two ratios of three quantities a, b and c are as, *a:b* =1 :2 and *b:c* = 2:3. Find their continued ratio.

**Solution:**

The ratios are:

$$a : b = 1 : 2$$
$$b : c = 2 : 3$$
$$a : b : c = ?$$

The common member of two ratios is b, so

$$\begin{array}{ccccc} a & & b & & c \\ 1 & : & 2 & & \\ & & 2 & : & 3 \\ \hline 1 & : & 2 & : & 3 \end{array}$$

Hence, *a:b:c* = 1:2:3

Therefore, 1:2:3 is the required continued ratio.

If the corresponding elements for the two ratios are not equal, then these are made equal by multiplying both the ratios by the numbers which make them equal as shown below.

$$\begin{array}{ccccc} a & : & b & & \\ & & c & : & d \\ \hline ac & : & bc & : & bd \end{array}$$

**Example 2:** The ratio of Saleem's income to Haider's is 2:3 and Imran's income to Saleem's is 1:5. Find the continued ratio among their incomes.

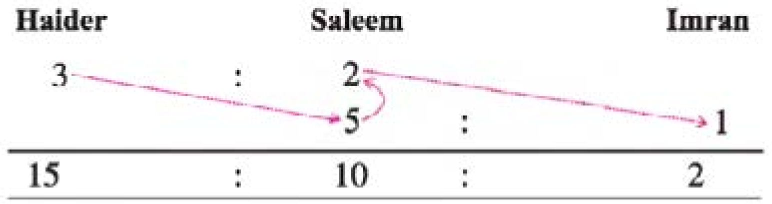
**Solution:**

The ratios are:

Saleem's income to Haider's = 2:3

Imran's income to Saleem's =1:5

Saleem is the common member but the value of his income is not the same in both ratios. Thus, first find the same values of common member as given below:



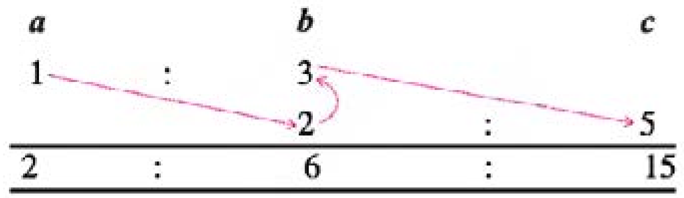

Thus, 15:10:2 is the required continued ratio.

**Example 3:** If *a:b* =1:3 and *b:c* = 2:5, then find *a:c*.

**Solution:**

The ratios are: *a:b* = 1:3, *b:c* = 2:5

We can see that b is the common member so,




Thus, *a:b:c* = 2:6:15

From the above, we can observe that the value of *a* = 2 and *c* = 15. So, *a : c* = 2:15.

## EXERCISE 6.1

1. If *a:b* = 3:5 and *b:c* = 5:6, then find *a:b:c*.
2. If *r:s* =1:4 and *s:t* = 2:3, then find *r:s:t*.
3. If *p:q* = 1:2 and *q:r* = 1:2, then find *p:q:r*.
4. If *x:z* =3:2 and *y:z* = 1:2, then find *x:y:z*.
5. If *l:m* = 1:7 and *l:n* = 5:6, then find *l:m:n*.
6. In a bakery, the ratio of the sale of bread to eggs is 2:3 and the sale of eggs to milk is 3:1. Find the continued ratio of bread, eggs and milk.
7. Ahmad and Irfan got a profit in a business in the ratio of 5:4 and Irfan and Waseem got a profit in the ratio of 8:9. Find the ratio of profit among Ahmad, Irfan and Waseem.





8. According to a survey, the people's liking for chicken and mutton are in the ratio of 2:1 and the people's liking for chicken and beef is in the ratio of 5:2.Find the ratio among people's liking for chicken, mutton and beef.
9. In a maths test Zara, Moona and Komal got marks in the ratio as given below:

   Zara to Moona = 4:5

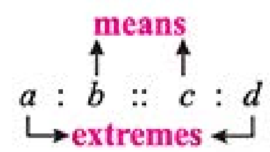
   Moona to Komal = 4:3

   Find continued ratio of marks obtained by Zara, Moona and Komal.

- **Proportion**

We have learnt in our previous classes that four quantities are said to be in proportion,if the ratio of the 1st to the 2nd is equal to the ratio of the 3rd to the 4th. In other words,the four quantities a,b,c and d are in proportion if *a:b* = *c:d.* Let us recap on what we studied in our previous class about proportion.

- In a proportion, the second and the third elements are called "means of a proportion" and the first and the fourth elements are called "extremes of a proportion" i.e.




- If second and third elements of a proportion have the same value such as:*a:b::b:c*

  Here 'b' is called mean proportional.
- One ratio is proportional to the other ratio, if and only if,

  product of means = product of extremes
- The 4th element of a proportion is known as the fourth proportional e.g., in proportion, *a* : *b* :: *c* : *d*, d is called the fourth proportional of *a*, *b* and *c*.
- A relation in which one quantity increases or decreases in the same proportion by increasing or decreasing the other quantity, is called the direct proportion.
- A relation in which one quantity increases in the same proportion by decreasing the other quantity and vice versa, is called inverse proportion.

- A method which is used to calculate the value of a number of things by finding the value of one (unit) thing is called the unitary method.

**Example 1:** Ghazi earns Rs.7500 in 2 weeks. What will he earn in 2 days if he works 6 days a week?

**Solution:**

**Unitary Method**

Ghazi earns in 12 days = Rs.7500 $\because$ 2 weeks =12days

Ghazi earns in 1 day $= \text{Rs.}\dfrac{7500}{12}$

Ghazi earns in 2 days. $= \text{Rs.}\dfrac{7500\times 2}{12} = \text{Rs.}1250$

Ghazi earns Rs.1250 in 2 days.

**Proportion Method**

Days are directly proportional to the rupees.

| Days | Rupees |
|---|---|
| ↑ 12 | ↑ 7500 |
| 2 | $x$ |

$$\frac{2}{12} = \frac{x}{7500}$$

$$x = \text{Rs.}\frac{2}{12}\times 7500 = \text{Rs.}1250$$

Ghazi earns Rs.1250 in 2 days.

**Example 2:** 10 boys complete a work in 4 days. In how many days will 20 boys complete the same work?

**Solution:**

**Unitary Method**

- More boys will complete the work in less number of days.
  10 boys complete the work = 4 days.
  1 boy complete the work = (4x10) days

20 boy complete the work = $\dfrac{4\times 10}{20}$ days = 2 days

**Proportion Method**

- Boys are inversely proportional to the days.





| Days | Boys |
|---|---|
| ↑ 4 | ↓ 10 |
| $x$ | 20 |

$$\frac{x}{4} = \frac{10}{20}$$

$$x = \frac{4\times 10}{20} = 2 \text{ days}$$

**Example 3:** 125 men can construct a road in 120 days. How many men can do the same work in 100 days?

**Solution:**

**Unitary Method**

- To do the work in less days, we need more men.
  In 120 days, men can construct the road = 125 men
  In 1 day, men can construct the road = (125 x 120) men

In 100 days, men can construct the road. $= \left(\dfrac{125\times 120}{100}\right)\text{men} = 150 \text{ men}$

**Proportion Method**

- Men are inversely proportional to the days.

| Men | Days |
|---|---|
| ↑ 125 | ↓ 120 |
| $x$ | 100 |

$$\frac{x}{125} = \frac{120}{100}$$

$$x = \frac{120\times 125}{100} = 150 \text{ men}$$

150 men can do the same work in 100 days.

## EXERCISE 6.2

1. Find the value of m in the following proportion.
   (i) 13:3 = m:6 (ii) m:5 = 3:10 (iii) 35:21 =5 :m
   (iv) 9:m = 54:42 (v) 0.21:6.3 = 0.06:m
   (vi) 1.1:m = 0.55:0.27
2. What is the fourth proportional of 2, 5 and 6?
3. Find mean proportional of 4 and 16.

4. A worker is paid Rs.2130 for 6 days. If his total wage during a month is Rs.9230,find the number of days he worked in the month.
5. Uzair takes 75 steps to cover a distance of 50m. How much distance will be covered in 375 steps?
6. If 2 boxes occupy a space of 500 $cm^3$, then how much space will be required for such 175 boxes?
7. An army camp of 200 men has enough food for 60 days. How long will the food last, if:
   a. The number of men is reduced to 160?
   b. The number of men is increased to 240?

## 6.2 Time, Work and Distance

### 6.2.1 Time and Work

- While solving the problems related to time and work, it can be observed that: time is directly proportional to work, because more work takes more time and less time gives less work.
- Number of workers is inversely proportional to the time, because more working hands take less time to complete a work whereas more time given for a work needs less working hands.

**Example 1:** If a girl can skip a rope 720 times in 1 hour. How many times can she skip in 35 minutes?

**Solution:**

Skipping a rope is directly proportional to the time. So, we can write this situation as;

| Time ↑ | Skip ↑ |
|---|---|
| 60 | 720 |
| 35 | $x$ |

∵ 1 hour = 60 minutes

$$\frac{35}{60}=\frac{x}{720}$$

$$x=\frac{720\times 35}{60}=420 \text{ times}$$





**Example 2:** If the heart of a human being beats 72 times in 1 minute. Find, in what time will the heart beat 204 times.

**Solution:**

(Heart beating is directly proportional to the time)

The situation can be written as:

| Time ↑ | Heart beat ↑ |
|---|---|
| 60 | 72 |
| $x$ | 204 |

∵ 1 min = 60 seconds

$$\frac{x}{60}=\frac{204}{72}$$

$$x=\frac{204\times 60}{72}=$$ 170 seconds or 2 min 50 sec

**Example 3:** If 36 men can build a wall in 21 days, find how many men can build the same wall in 14 days.

**Solution:**

Men are inversely proportional to the time. So, this situation can be written as:

| Men ↑ | Days ↓ |
|---|---|
| 36 | 21 |
| $x$ | 14 |

(inverse proportion)

$$\frac{x}{36}=\frac{21}{14}$$

$$x=\frac{21\times 36}{14}=54 \text{ men}$$

## EXERCISE 6.3

1. If a man can weave 450m cloth in 6 hours. How many metresof cloth can he weave in 14 hours?
2. If a 162km long road can be constructed in 9 months. How many number of months are required to construct a 306 km long road ?

3. 540 men can construct a building in 7 months. How many men should be removed from work to finish the building in 9 months?
4. Asma can iron 5 shirts in 14 minutes. How long will she take to iron 35 shirts?
5. 12 water pumps can make a water tank empty in 20 minutes. But 2 pumps are out of order. How long will the remaining pumps take to make the tank empty?
6. 14 horses graze a field in 25 days. In how many days will 35 horses graze it?
7. A mason can repair a 744m long track in 24 days. If he repairs 589m track, then find how many days will he take to repair the remaining track.
8. A farmer can plough an area of 40 acres in 16 hours. How many acres will he plough in 36 hours?
9. A dish washer deems 1350 dishes in 1 hour. How many dishes will it wash in 16 more minutes?

## 6.2.2 Relation between Time and Distance

In our daily life, we observe many moving things like vehicle, birds, human beings, ships, animals, etc. in our surroundings. While moving these things cover a certain distance in a certain time at a certain speed. To understand the relation between these three quantities we can use a formula which is given below:

Distance = Speed x Time

From the given formula, it can be examined that:

- Distance is directly proportional to the time and speed.
- Time is inversely proportional to the speed.

| An interval between two happenings is called time. Its basic unit is second. |
|---|

- **Units of Speed**

"The distance covered per unit time is called speed."

Speed is measured in different units that is, kilometres per hour, metres per second, etc. We write these units by dividing the units of distance (km, m) by the units of time.





(hr, min, sec).

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The units of speed are mutually convertable. Let us make it clear with the help of some examples.

**Example 1:** Convert the speed of 54 kilometers per hour into metres per second.

**Solution:**

Speed = 54 km/hour ∵ km = 1000 m
Distance = 54 km ∵ 1 hour = 60 min
Time = 1 hour 1 min = 60 sec

$$\text{Speed } \frac{54\times 1000}{60\times 60}$$
$$= \frac{54000}{3600} = 15 \text{ metre / second}$$

**Example 2:** Convert the speed of 10 meters per second into kilometres per hour.

**Solution:**

Speed = 10 m/sec ∵ 1000m = 1km

Distance = 10 meter $1\text{m} = \frac{1}{1000}\text{km}$

Time = 1 second ∵ 3600 sec = 1 hour

$1 \text{ sec} = \frac{1}{3600}\text{hour}$

$$\text{Speed} = \frac{10/1000}{1/3600}$$
$$= \frac{10\times 3600}{1\times 1000} = 36 \text{ km/hour}$$

**Example 3:** A truck covers a distance of 360 kilometres in 5 hours. Find its speed in:

(i) kilometres per hour (ii) metres per second

**Solution:**

Distance = 360 km, Time = 5 hours, Speed = ?

(i) kilometres per hour

By using the formula,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{360}{5} = 72 \text{ km/hour}$$

(ii) metres per second

Distance in metres = 360 x 1000 = 360000m

Time in seconds = 5 x 60 x 60 = 18000 sec

Now change the unit of speed into metres per second.

Speed = 72 km/hour

$$= \frac{72 \times 1000}{1 \times 60 \times 60} = \frac{72000}{3600} = 20 \text{ metre/second}$$

## EXERCISE 6.4

1. Convert the unit of speed into metres per second.
   (i) 72 km/hour (ii) 144 km/hour (iii) 216 km/hour
   (iv) 360 km/hour (v) 180 km/hour (vi) 1152 km/hour
2. Convert the unit of speed into kilometres per hour.
   (i) 10 m / sec (ii) 25 m / sec (iii) 5 m / sec
   (iv) 15 m / sec (v) 30 m / sec (vi) 20 m / sec
3. Iram walks up to her school at a speed of 4 km/hour. It takes 45 minutes to reach the school. How far is her school from her home?
4. A non-stop train leaves Lahore at 4:00 p.m and reaches Karachi at 10:00 a.m next day. The speed of the train was 70 km / hour. Find the distance between Lahore and Karachi.
5. A cyclist crosses a 30 metre long bridge in $3\frac{1}{4}$ minutes. Find the speed of the cyclist.
6. A car covers 201 kilometres in 3 hours. How much distance will it cover in 7 hours?
7. A truck moves at the speed of 36 kilometres per hour. How far will it travel in 15 seconds?





8. A bus leaves Islamabad at 11:00 a.m and reaches Lahore at 3:00 p.m. If the distance between Lahore and Islamabad is 380 km,find the speed of the bus.

## REVIEW EXERCISE 6

1. Answer the following questions.
   (i) Define direct proportion.
   (ii) What is continued ratio?
   (iii) Write the formula to show the relation between time, speed and distance.
   (iv) Define speed.

2. Fill in the blanks.
   (i) Distance is directly proportional to the ______ and speed.
   (ii) Number of workers is ______ proportional to the time.
   (iii) The combination of two ratios of three quantities is called a ______ ratio.
   (iv) Distance = ______ x time.
   (v) $\text{Speed} = \frac{\square}{\text{Time}}$
   (vi) In two ratios *a : b* and *b : c*, *b* is called the______ .

3 Tick ( ✓ ) the correct answer.

4. Find the missing terms in the table, if is directly proportional to the y.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 4 | | 12 | 16 | |

5. Find the missing terms in the table, if $x$ is inversely proportional to $y$.

| $x$ | 1 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | 24 | 12 | | | 3 |

6. In a class, 8 ice creams are served for every group of 5 students. How many ice creams will be served if there are 40 students in the class?
7. In a hostel of 50 girls, there are food provisions for 40 days. If 30 more girls join the hostel, how long will the provisions last?

8. How many days will 1648 persons take to construct a bridge, if 721 persons can build the same in 48 days?

9. A rickshaw travels at the speed of 36km per hour. How much distance will it travel in 20 seconds.

10. A bus covers a distance in 3 hours at a speed of 60km per hour. How much time will it take to cover the same distance at a speed of 80km per hour?

## SUMMARY

- Two ratios of three quantities can be combined into a continued ratio to express the relation of these quantities.
- A relation in which one quantity increases or decreases in the same proportion by increasing or decreasing the other quantity, is called the direct proportion.
- A relation in which one quantity increases in the same proportion by decreasing the other quantity and vice versa, is called the inverse proportion.
- Time is directly proportional to the work, and the number of workers is inversely proportional to time.
- To understand the relation between distance, speed and time, we use the formula:

  Distance = Speed x Time
- An interval between two happenings is called time.
- The distance covered per unit time is called speed.

CHAPTER

# 7 FINANCIAL ARITHMETIC

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Explain property tax and general sales tax.
- Solve tax-related problems.
- Explain Profit and markup.
- Find the rate of profit/markup per annum.
- Solve real life problems involving profit/markup.
- Define zakat and ushr.
- Solve problems related to zakat and ushr.

# 7.1 Taxes

Government needs money to run a state. For this purpose, government collects an amount from the public and provides them facilities like security, hospitals, education, defense, roads, parks, etc. This amount is called a tax. We pay different types of taxes in Pakistan but here we shall, discuss only property and general sales tax.

### 7.1.1 Property Tax and General Sales Tax

- **Property Tax**

The tax which is received on a property is called the property tax. Property tax is a provincial tax paid on the value of a property. It is generally paid at a flat rate of 2% but the tax rates vary, depending on the province.

**Example 1:** Find the property tax on a property of Rs.6,200,000 at the rate of 0.8%.

**Solution:**

Worth of the property = Rs.6,200,000
Tax rate = 0.8%
Property tax = ?
Property tax = 0.8% of Rs.6,200,000





$$= \frac{8}{1000} \times 6{,}200{,}000$$

= Rs.49,600

**Example 2:** Raheem paid Rs. 8,676 as a property tax at the rate of 2%. Find the worth of Raheem's property.

**Solution:**

Property tax = Rs. 8,676
Tax rate = 2%
Worth of the property = ?

By using the unitary method

2% of the worth of property = Rs. 8,676

1% of the worth of property $= \text{Rs.} \left(\frac{8{,}676}{2}\right)$

100% of the worth of property $= \text{Rs.} \left(\frac{8{,}676}{2} \times 100\right) = \text{Rs. } 433{,}800$

The worth of Raheem's property is Rs. 433,800.

- **General Sales Tax**

"The tax a buyer pays to the seller at the time of buying things is called general sales tax". General sales tax is imposed by the government on the percentage of the selling prices of things. In Pakistan, its rate varies from 0% to 25% depending on exemptions and types of industry.

| |
|---|
| In Pakistan some basic items including wheat, rice, pulses, vegetables, meat, poultry, books, drugs, etc. are exempted from the general sales tax. |

**Example 3:** Saleem bought a car for Rs. 875,000 and paid 16% as a tax. How much tax did he pay?

**Solution:**

Price of the car = Rs. 875,000
Tax = 16%
General sales tax = ?

| Remember |
|---|
| The standard rate of sales tax in Pakistan is 16%. |

GST = 16% of Rs.875,000

$$= \frac{16}{100} \times 875{,}000$$

= (16 x 8750) = Rs. 140,000

**Example 4:** The price of a mobile is Rs.8,800 inclusive of a 10% GST. What is the original price of the mobile?

**Solution:**

Price of the mobile = Rs.8,800
GST rate = 10%
Original price = ?
Price % of the mobile = 100% + 10% = 110%

By using the unitary method

110% price of the mobile = Rs.8,800

$$\text{1\% price of the mobile} = \text{Rs.}\left(\frac{8{,}800}{110}\right)$$

$$\text{100\% price of the mobile} = \text{Rs.}\left(\frac{8{,}800}{110} \times 100\right) = \text{Rs. } 8{,}000$$

Thus, the original price of the mobile is Rs.8,000.

## EXERCISE 7.1

1. Calculate the price that a customer has to pay for each article with a 16% general sales tax imposed on it.
   (i) Football = Rs.800 (ii) Rackets = Rs. 1,250
   (iii) Hockey = Rs. 1,650 (iv) Bat = Rs.2,100
2. Find the property tax on a property of worth Rs.948,000 at the rate of 1.5%.
3. Haris paid the property tax of Rs.2,068 at the rate of 0.8%. Find the worth of property.
4. Property tax Rs. 18,720 was paid when the worth of property is Rs. 1,560,000. Find the percentage of property tax.
5. The price of a toy including 5% general sales tax is Rs.945. Find the original price of the toy.





6. Find the property tax on a property of Rs.650,000 at the rate of 1.8%.
7. Farah paid the property tax of Rs. 9,240 at the rate of 2%. Find the worth of her property.
8. The price of a bicycle is Rs.6,480 inclusive 8% GST. What is the original price of the bicycle?

## 7.2 Profit and Markup

We know that in a business, generally goods are bought at a certain price and sold at a higher price. In such a case, there is a gain, i.e.

Sale price – Cost price = Gain

While discussing this gain, we often use two different terms, profit and markup. To understand the difference between these two terms, let us learn them one by one.

- **Profit**

A profit means what we have earned after selling a thing. It is calculated as a percentage of the cost price as shown below.

$$\text{Profit\%} = \frac{\text{Gain}}{\text{Cost price}} \times 100\%$$

- **Markup**

In our daily life, we often borrow money from our friends and relatives to buy a thing that we repay them after a certain period. Some banks and retail organizations also provide the same services and charge an additional amount called markup.

“A markup is an amount added to a cost price to calculate the sale price.”

Usually, we calculate the markup as a percentage of the actual amount paid for a thing. This is called the markup rate and paid amount itself is called the principal. Suppose “P” is the principal, “T” is the time period and “R” is the markup rate, then the amount of markup will be:

$$\text{Markup} = \frac{\text{RPT}}{100}$$

**Example 1:** Ada bought a jewelry set for Rs.84,000 and sold for Rs.855,00. Find the percentage of profit.

**Solution:**

Cost price (C.P) = Rs.84,000
Sale price (S.P) = Rs.85,500
Gain = Sale price – Cost price
= Rs.85,500 – Rs.84,000 = Rs. 1,500

$$\%\text{Profit} = \frac{\text{Gain}}{\text{Cost price}} \times 100\%$$

$$= \left(\frac{1{,}500}{84{,}000} \times 100\right)\% = 1.79\%$$

**Example 2:** Aleem bought a television for Rs. 15,000 on installments at the markup rate of 12% per annum. Find the selling price of the television if time period is 3 years.

**Solution:**

Cost price (P) = Rs. 15,000 ; Markup rate = 12% per annum
Time period (T) = 3 years ; Price of the Television = ?

Using the formula,

$$\text{Amount of the markup} = \frac{\text{RPT}}{100} = \frac{12 \times 15{,}000 \times 3}{100} = \text{Rs. } 5{,}400$$

Price of the television = cost price + markup
= Rs. 15,000 + Rs. 5,400 = Rs. 20,400

**Example 3:** Imran sold a bicycle for Rs. 3,978 and got 17% profit. Find the cost price of the bicycle.

**Solution:**

Sale price (S.P) = Rs.3,978
% Profit = 17%

By using formula,





$$\text{Cost price (C.P)} = \frac{\text{Sale price}}{(100\% + \text{Profit}\%)}$$

$$\text{Cost price (C.P)} = \left(\frac{3978}{117} \times 100\right) = \text{Rs. } 3{,}400$$

**Example 4:** Hatim bought a bike for Rs. 135,000 and sold at 62% profit. Find the sale price of the bike.

**Solution:**

**Method I**

% profit = 62%
Cost price = Rs.135,000
Sale price = ?
Sale price = (100% + 62%) x 135,000 rupees
= 162% x 135,000 rupees

$$= \left(\frac{162}{100} \times 135{,}000\right) = \text{Rs. } 218{,}700$$

**Method II**

$$\%\text{Profit} = \frac{\text{Profit}}{\text{Cost price}} \times 100$$

$$62 = \frac{\text{Profit}}{135{,}000} \times 100$$

$$\text{Profit} = \left(\frac{62 \times 135{,}000}{100}\right)\text{rupees}$$

$$= \text{Rs. } 83{,}700$$

We know that;

Sale price = cost price + profit
= Rs.135,000 + Rs.83,700
= Rs.218,700

**Example 5:** Find the markup on a thing whose price is Rs. 45,000 for 73 days at the rate of 10% per annum.

**Solution:**

Principal (P) = Rs. 45,000, Markup rate (R) = 10% per annum, Markup = ?

$$\text{Time period} = 73 \text{ days} = \frac{73}{365}\text{year} = \frac{1}{5}\text{year}$$

$$\text{By using the formula: Markup} = \frac{RPT}{100}$$

$$= \text{Rs.}\ \frac{10\times 45{,}000\times\frac{1}{5}}{100}$$

$$= \text{Rs.}\ \frac{10\times 45{,}000\times 1}{100\times 5} = \text{Rs. } 900$$

**Example 6:** The markup on a principal amount is Rs. 820 for 6 months at the rate of 12.5% per annum. Calculate the principal amount.

**Solution:**

Markup = Rs. 820    Markup rate (R) = 12.5%

$$\text{Time period (T)} = 6 \text{ months} = \frac{6}{12}\text{year} = \frac{1}{2}\text{year}$$

Principal amount (P) = ?

$$\text{By using the formula: Markup} = \frac{RPT}{100}$$

$$820 = \frac{12.5\times\frac{1}{5}\times P}{100}$$

$$\text{Principal amount} = \frac{820\times 100\times 2}{12.5\times 1} = \text{Rs. } 13{,}120$$

## EXERCISE 7.2

1. Find the missing quantities by using the formula





| | Markup | Principal | Time Period | Markup rate |
|---|---|---|---|---|
| (i) | | Rs. 500 | 2 years | 12% |
| (ii) | Rs. 205 | | 1 year | 8% |
| (iii) | Rs. 528 | Rs. 1,650 | 10 years | |
| (iv) | Rs. 350 | Rs. 3,500 | | 2.5% |
| (v) | | Rs. 100,000 | 3 years | 1.25% |
| (vi) | Rs. 1,050 | | 5 years | 4.5% |

2. Adnan bought 96 eggs at the rate of Rs.40 per dozen and sold at the rate of Rs.4 per egg. Find the percentage of profit, if 3 eggs were rotten.
3. If 16% profit on a mobile set is Rs.832. Find the cost price of the mobile set.
4. Zia bought an out of order clock for Rs.750 and got it repaired for Rs.425. What should be the selling price of the clock if Zia wants to earn 25% profit?
5. Find the markup on a principal amount of Rs. 75,500 at the rate of 9% per annum for 4 years.
6. Ujala bought a car for Rs.280,000 and spent Rs. 12,000 more on it. What should be the selling price if she wants to get 7.5% profit?
7. The price of a bicycle including markup is Rs. 5610. If the markup rate is 5% per annum, find the amount of markup for 146 days.
8. Khushi bought a computer for Rs. 100,000 and paid a markup of Rs. 25,000 for 2 years. What markup rate did she pay?

## 7.3 Zakat and Usher

Zakat and Ushr are levied as ordered in the Holy Qur'an and Sunnah. Let us discuss them one by one.

- **Zakat**

Zakat is one of the five pillars of Islam which is ordrs by Almighty Allah which is paid on the wealth which remains with a person for a complete year. Islam has fixed its rate, that is 2.5%.

| |
|---|
| Nisab (minimum limit of wealth that attracts liability of Zakat) in case of gold is 7.5 tolas and in case of silver is 52.5 tolas. |

**Example 1:** Calculate the amount payable as Zakat by Haleem who saves Rs.949,000 for one year.

**Solution:**

Total Saving = Rs.949,000
Rate of Zakat = 2.5%
Amount of Zakat = ?
Amount of Zakat = 2.5% of Rs.949,000

$$= \left(\frac{2.5}{100} \times 949{,}000\right) \text{rupees}$$

$$= \left(\frac{25 \times 949{,}000}{1{,}000}\right) \text{rupees} = \text{Rs. } 23{,}725$$

Thus, Haleem will pay Rs.23,725 as Zakat.

**Example 2:** Find the wealth of Ibrahim if he paid Rs.7,500 as Zakat.

**Solution:**

2.5% of Ibrahim's wealth = Rs.7,500

$$1\ \%\ \text{of Ibrahim} = \text{Rs. } \frac{7{,}500}{2.5}$$

$$100\ \%\ \text{of Ibrahim} = \text{Rs. } \frac{7{,}500}{2.5} \times 100 = \text{Rs.}300{,}000$$

- **Ushr**

Ushr means one-tenth. It is paid on agricultural products. It is paid at the rate of 10% of the produce in case a piece of land irrigated by natural sources like rain, springs, streams, etc. However, the rate of Ushr is one-half, i.e. 5% of the entire produce in case





a piece of land watered by artificial means of irrigation such as wells, buckets, tube well, etc.

| |
|---|
| Nisab (minimum amount of agricultural produce) which is liable to Ushr is 948kg in weight. If the produce is less than that, no Ushr is chargeable. |

**Example 3:** A farmer sold his crop of wheat for Rs.995,400. Find the amount of Ushr at the rate of 10%.

**Solution:**

Total Amount = Rs.995,400
Rate of Ushr = 10%
Amount of Ushr = ?
Amount of Ushr = 10% of Rs.995,400

$$= \left(\frac{10}{100} \times 995{,}400\right) = \text{Rs. } 99{,}540$$

Thus, amount of Ushr is Rs.99,540

**Example 4:** Adnan sold mangoes and paid Rs. 3,675 as the amount of Ushr at the rate of 5%. Find the sale price of the mangoes.

**Solution:**

Amount of Ushr = Rs.3,675
Rate of Ushr = 5%
Amount of mangoes = ?
5% of mangoes amount = Rs. 3,675

$$1\%\ \text{of mangoes amount} = \text{Rs.}\left(\frac{3{,}675}{5}\right)$$

$$\text{Price of mangoes} = \text{Rs.}\left(\frac{3{,}675}{5} \times 100\right)$$

Price = Rs. 73,500

Thus, the amount of mangoes is Rs.73,500

## EXERCISE 7.3

1. An amount of Rs.62,480 remained with Nosheen for a complete year. How much Zakat will she pay?
2. Saba paid Rs.2,250 as Zakat. What is the worth of her wealth?
3. Nadeem paid Rs.6,075 as Zakat. How much wealth did he have?
4. Saleem earned Rs. 114,700 from a rice crop and paid Ushr at the rate of 5%. What amount did he pay as Ushr?
5. Nabeel sold apples for Rs.398,160 and paid 10% as Ushr. Find the amount of Ushr.
6. Shama's annual saving is Rs. 222,000. What is the amount of Zakat to be paid by her?
7. Nahal paid Rs.7,895 as Ushr at the rate of 10%. What amount did she earn?
8. Calculate the amount payable as Ushr by a farmer who earned Rs.88,460. Find the actual amount, if rate of Ushr is 5%.

## Review Exercise 7

1. Answer the following questions.
   (i) What is meant by the tax?
   (ii) Define the general sales tax.
   (iii) What is the difference between profit and markup?
   (iv) What rate of Zakat has Islam fixed?
   (v) What is Ushr?
2. Fill in the blanks.
   (i) The tax which is received on a property is called the_______.
   (ii) The tax a buyer pays to the seller at the time of buying things is called _______.
   (iii) An amount added to a cost price to calculate the sale price is called a _______.
   (iv) Zakat and Ushr are levied as ordered in the____ and Sunnah.
   (v) A markup is an amount added to _______to calculate the sale price.





3. Tick (✓) the correct answer.
4. Calculate the amount of property tax of a house at the rate of 2%. The value of the house is Rs. 1,450,000.
5. Adnan has paid Rs. 16,000 as a property tax at the rate of 1.6%. Find the value of his property.
6. The price of a toy is Rs.500. Find the sale price of the toy if GST is 16%.
7. Nabeel bought a bag for Rs.4,000 and paid Rs.560 more as GST. Find the percentage of GST.
8. Nadeem sold a bicycle for Rs.4,500 with markup of 25%. Find the cost price of bicycle.
9. A shopkeeper sold a calculator for Rs.900 and earned 22% profit. Find the actual price of a calculator.
10. Komal saves Rs.96,000 in a year. How much will she pay as Zakat?
11. Saleem has 2,400kg wheat. The price of wheat is Rs.30kg. Find the Ushr that he will pay.

## Summary

- A tax is a fee charged on the public at the rate fixed by a government to run its affairs.
- The tax which is received on property is called the property tax.
- The tax a buyer pays to the seller at the time of buying things is called general sales tax.
- An amount added to a cost price to calculate the sale price is called a markup.
- A profit means what we have earned after selling a thing.
- Zakat is one of the five pillars of Islam which is ordrs by Almighty Allah which is paid on the wealth which remains with a person for a complete year. Islam has fixed its rate, that is 2.5%.
- Ushr means one-tenth. It is paid on agricultural products.

CHAPTER

# 8 ALGEBRAIC EXPRESSIONS

*Animation 8.1: Algebraic expression*
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## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Define a constant as a symbol having a fixed numerical value.
- Recall a variable as a quantity which can take various numerical values.
- Recall a literal as an unknown number represented by a letter of an alphabet.
- Recall an algebraic expression as a combination of constants and variables connected by the sign of fundamental operations.
- Define a polynomial as an algebraic expression in which the powers of variables are all whole numbers.
- Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms and three terms respectively.
- Add two or more polynomials.
- Subtract a polynomial from another polynomial.
- Find the product of:
  1. monomial with monomial.
  2. monomial with binomial/trinomial.
  3. binomials with binomial/trinomial.
- Simplify algebraic expressions involving addition, subtraction and multiplication.
- Recognize and verify the algebraic identities:
  1. $(x + \text{a})(x + b) = x^2 + (a + b)x + ab$,
  2. $(a + b)^2 = (a + b)(a + b) = \text{a}^2 + 2ab + b^2$,
  3. $(a – b$ f = $( a –b ) \{ a – b ) = \text{a} – 2ab + b^2$,
  4. $a^2 – b^2 = (a - b)(a + b)$.
- Factorize an algebraic expression (using algebraic identities).
- Factorize an algebraic expression (making groups).

# 8.1 Algebraic Expressions

Algebra is one of the useful tools of mathematics. It uses mathematical statements to describe the relationships between things that vary over time. In our previous class, we have learnt the





introduction to the basic ideas of algebra including the effects of some basic operations, concept of variables and simplification of an algebraic expression with its evaluation.

| Do you Know |
| --- |
| **Algebra is an Arabic word which means “bringing together broken parts”.** |

### 8.1.1 Literals

The letters or alphabets that we use to represent unknowns are called literal numbers. For example, area of a rectangle can be calculated by multiplying its length and breadth, i.e.

$$\text{Area} = l \times b$$

Where, $l$ = length and $b$ = breadth. Clearly, $l$ and $b$ represent the unknowns. So, these are called *literal* numbers.

### 8.1.2 Constant

A symbol having a fixed numerical value is called a constant. For example, 2, 7, 11, etc. are all constants.

### 8.1.3 Variable

A symbol represented by a literal and can take various numerical values is called a variable, i.e. in $x + 1$, $x$ is a variable and 1 is a constant.

### 8.1.4 Algebraic Expressions

A combination of constants and variables connected by the signs of fundamental operations (+, ÷, –, x) is called an algebraic expression, i.e. 8, $4x + y$, $x^2 + y^2$, $a^2 – 2ab + b^2$, etc.

- **Algebraic Terms**

The parts of an algebraic expression separated by the operational signs "+" and "–" are called its terms, i.e. in $x + y$, $x$ and $y$ are its two terms.

### 8.1.5 Polynomial

Normally, the word poly is used for more than one things but in algebra polynomial represents an algebraic expression containing a single term as well as two or more than two terms.
For a polynomial, the exponents of the variables must be the whole numbers. For instance, 9, $3x$, $x^2 + 2$, $x^3 + 2x + 1$, etc. all expressions are polynomials but $x^{-2} + 1$, $x^{1/2} + 3x + 2$, etc. are not polynomials because their exponents (–2 ; 1/2) are not whole numbers.
"An algebraic expression in which the exponents of variables are all whole numbers is called a polynomial".

### 8.1.6 Identification of a monomial, bionomial and trinomial

**Monomial:** A polynomial having one term is called a monomial, i.e. 5, $3x$, $2ab$, etc. are monomials.
**Binomial:** A polynomial having two terms is called a binomial, i.e. $6x + a$, $a - 3b$, etc. are binomials.
**Trinomial:** A polynomial having three terms is called a trinomial, i.e. $x^2 + 3x + 5$, $2a + 3b + c$, etc. are trinomials.

| |
|---|
| In routine, we write a polynomial in descending order and arrange a polynomial with respect to one variable, e.g. we arrange the polynomial $x^3y^2 + y^4 + x^4 - x^2y^3$ with respect to $x$ as, $x^4 + x^3y^2 - x^2y^3 + y^4$. |

## EXERCISE 8.1

1. Add the terms to write an algebraic expression.
   (i) $2ab, 3bc, ca$ (ii) $7l^2, 3m^2, -8$ (iii) $p^2, -q^2, -r^2$
   (iv) $5xyz, 2yz, -8xy$ (v) $-2ab, a, -bc, c$
   (vi) $9lm, 8mm, -10ml, -2$





2. Write constants and variables used in each expression.
   (i) $x + 3$ (ii) $3a + b - 2$ (iii) $l^2 + m^2 + n^2$
   (iv) $5a$ (v) $2x^2 - 1$ (vi) $3l^2 - 4n^2$
3. Identify monomials, binomials and trinomials.
   (i) $x + y - z$ (ii) $-6l$ (iii) $2x^2 - 3$
   (iv) $abc$ (v) $x^2 + 2xy + y^2$ (vi) $(-a)^3$
   (vii) $l - m$ (viii) $7a^2 - b^2$ (ix) $lm + mn + nl$
   (x) $2a - 3b - 4c$ (xi) $11x^2y^2$ (xii) $a^3 + a^2b + ab^2$

## 8.2 Operations with Polynomials

Recall that in our previous class, we have learnt the application of some basic operations in algebra. Now we learn more about them.

### 8.2.1 Addition and Subtraction of Polynomials

In polynomials, we use the same method for addition and subtraction that we use for like terms, which is given below.

- We can arrange the polynomials in any order but usually we arrange them in descending order and write the like terms vertically in a single column for adding.
- For subtraction, we just change the signs of the terms of the polynomial which are to be subtracted and simply add them.

**Example 1:** Add the following polynomials.
(i) $2x^4y^2 + x^3y + x^2y - 5$, $2x^2y - x^4y^2 + x^3y + 1$, $2 - x^4y^2 + x^3y - 7x^2y$
(ii) $x^2 + y^2 + 2xy$, $y^2 + z^2 + 2yz$, $2x^2 + 3y^2 + z^2$, $z^2 - 2xy - 2yz$

**Solution:**
(i) $2x^4y^2 + x^3y + x^2y - 5$, $2x^2y - x^4y^2 + x^3y + 1$, $2 - x^4y^2 + x^3y - 7x^2y$
Arrange the polynomials in descending order and write all like terms in a single column.

$2x^4y^2 + x^3y + x^2y - 5$
$-x^4y^2 + x^3y + 2x^2y + 1$
$-x^4y^2 + x^3y - 7x^2y + 2$
$0\,x^4y^2 + 3x^3y - 4x^2y - 2$

- $2x^4y^2 - x^4y^2 - x^4y^2 = (2-1-1)x^4y^2 = 0x^4y^2$
- $x^3y + x^3y + x^3y = (1+1+1) = 3x^3y$
- $x^2y + 2x^2y - 7x^2y = (1 + 2 - 7) = -4x^2y$
- $-5 + 1 + 2 = -2$

Thus $x^4y^2 + 3x^3y - 4x^2y - 2$ is the required polynomial.

(ii) $x^2 + y^2 + 2xy,\ y^2 + z^2 + 2yz,\ 2x^2 + 3y^2 + z^2,\ z^2 - 2xy - 2yz$

Arrange the polynomials in descending order and write all like terms in a single column.

$x^2 + y^2 + 2xy$
$y^2 + z^2 + 2yz$
$2x^2 + 3y^2 + z^2$
$z^2 - 2xy - 2yz$
$3x^2 + 5y^2 + 3z^2 + 0xy + 0yz$

- $x^2 + 2x^2 = (1 + 2)\,x^2 = 3x^2$
- $y^2 + y^2 + 3y^2 = (1 + 1 + 3)\,y^2 = 5y^2$
- $z^2 + z^2 + z^2 = (1+1+1)z^2 = 3z^2$
- $2xy - 2xy = (2 - 2)xy = 0$
- $2yz - 2yz = (2 - 2)\,yz = 0$

Thus, $3x^2 + 5y^2 + 3z^2$ is the required polynomial.

**Example 2:** What should be added to $3 + 2x - x^3y^2 + 4x^2y$ to get $2x^3y^2 + x^2y - 3x - 1$?

**Solution:**

Arrange the polynomials in descending order.

1st polynomial $= 2x^3y^2 + x^2y - 3x - 1$

2nd polynomial $= -x^3y^2 + 4x^2y + 2x + 3$

If we subtract the 2nd polynomial from 1st polynomial, we can get the required polynomial.

$2x^3y^2 + x^2y - 3x - 1$
$\mp x^3y^2 \pm 4x^2y \pm 2x \pm 3$
$3x^3y - 3x^2y - 5x - 4$

- $2x^3y^2 + x^3y^2 = (2+1) = 3\,x^3y^2$
- $x^2y - 4x^2y = (1 - 4) = -3x^2y$
- $-3x - 2x = (3 - 2)x = -5x$
- $-1 - 3 = -4$

Thus, $3x^3y - 3x^2y - 5x - 4$ is the required polynomial.

**Example 3:** What should be subtracted from $3x^4y^2 + 11 + 4x^6y^4 - 6x^2y$ to get $1 + x^4y^2 - x^2y + x^6y^4$?

**Solution:**

Arrange the polynomials in descending order.

1st polynomial $= 4x^6y^4 + 3x^4y^2 - 6x^2y + 11$

2nd polynomial $= x^6y^4 + x^4y^2 - x^2y + 1$





If we subtract the 2nd polynomial from 1st polynomial, we can get the required polynomial.

$4x^6y^4 + 3x^4y^2 - 6x^2y + 11$
$\pm x^6y^4 \pm x^4y^2 \mp x^2y \pm 1$
$3x^6y^4 + 2x^4y^2 - 5x^2y + 10$

- $4x^6y^4 - x^6y^4 = (4 - 1) = 3x^6y^4$
- $3x^4y^2 - x^4y^2 = (3 - 1) = 2x^4y^2$
- $-6x^2y + x^2y = (-6+1) = -5x^2y$
- $11 - 1 = 10$

Thus, $3x^6y^4 + 2x^4y^2 - 5x^2y + 10$ is the required polynomial.

## EXERCISE 8.2

1. Add the following polynomials.
   (i) $x^2 + 2xy + y^2,\ x^2 - 2xy + y^2$
   (ii) $x^3 + 3x^2y - 2xy^2 + y^3,\ 2x^3 - 5x^2y - 3xy^2 - 2y^3$
   (iii) $a^5 + a^3b - 2ab^3 + b^3,\ 4a^5 + 3a^3b + 2ab^3 + 5b^3$
   (iv) $2x^4y - 4x^3y^2 + 3x^2y^3 - 7xy^4,\ x^4y - 4x^3y^2 - 3x^2y^3 + 8xy^4$
   (v) $ab^5 + 12a^2b^4 - 6a^3b^3 + 10a^4b^2 - a^5b,\ 4ab^5 - 8a^2b^4 + 6a^3b^3 - 6a^4b^2 + 4a^5b$
2. If $A = x - 2y + z$, $B = -2x + y + z$ and $C = x + y - 2z$ then find.
   (i) $A - B$ (ii) $B - C$ (iii) $C - A$
   (iv) $A - B - C$ (v) $A + B - C$ (vi) $A - B + C$
3. What should be added to $x^7 - x^6 + x^5 - x^4 + x^3 - x^2 + x + 1$ to get $x^7 + x^5 + x^3 - 1$?
4. What should be added to $2x^4y^3 - x^3y^2 - 3x^2y - 4$ to get $5x^4y^3 + 2x^3y^2 + x^2y - 9$?
5. What should be subtracted from $5x^5y^5 - 3x^3y^3 + 10xy - 9$ to get $3x^5y^5 + 7x^3y^3 - 11xy + 19$?

### 8.2.2 Multiplication of Polynomials

While multiplying two polynomials in addition to the commutative, associative and distributive laws, we also use the laws of exponents that can be seen in the given examples

- **Multiplying monomial with monomial**

**Example 1:** Find the product of:

(i) $4a^2$ and $5a^3$ (ii) $5x^2$ and $3y^2$ (iii) $3l^4m^2n$ and $7l^5m^8n^6$

**Solution:**

(i) $4a^2$ and $5a^3$

$4a^2 \times 5a^3 = (4 \times 5)(a^2 \times a^3)$

$= (20)(a)^{(2+3)}$ $\because$ product law

$= 20a^5$ $a^m \times a^n = a^{m+n}$

(ii) $5x^2$ and $3y^2$

$5x^2 \times 3y^2 = (5 \times 3)(x^2 \times y^2)$

$= (15)(x^2y^2)$

$= 15x^2y^2$

(iii) $3l^4m^2n$ and $7l^5m^8n^6$

$3l^4m^2n \times 7l^5m^8n^6 = (3 \times 7)\ (l^4 \times l^5)\ (m^2 \times m^8)\ (n \times n^6)$

$= 21 \times l^{4x5} \times m^{2+8} \times n^{1+6} = 21l^9m^{10}n^7$

- **Multiplying monomial with Binomial / Trinomial**

**Example 2:** Simplify:

(i) $3x^2(x^2 - y^2)$ (ii) $-6a^2\ (2a + 3b)$

(iii) $2l^2m^2n^2\ (3lm - 2mn + 5nl)$

**Solution:**

(i) $3x^2(x^2 - y^2)$

$= (3x^2 \times x^2) - (3x^2 \times y^2)$

$= 3(x^{2+2}) - 3(x^2 \times y^2)$

$= 3x^4 - 3x^2y^2$

(ii) $-6a^2\ (2a + 3b)$

$= (-6a^2 \times 2a) + (-6a^2 \times 3b)$

$= (-6 \times 2)\ (a^2 \times a) + (-6 \times 3)(a^2 \times b)$

$= (-12)(a^{2+1}) + (-18)\ (a^2b)$

$= -12a^3 - 18a^2b$

(iii) $2l^2m^2n^2\ (3lm - 2mn + 5nl)$

$= (2l^2m^2n^2 \times 3lm) - (2l^2m^2n^2 \times 2mn) + (2l^2m^2n^2 \times 5nl)$

$= (2 \times 3)\ (l^2m^2n^2 \times lm) - (2 \times 2)\ (l^2m^2n^2 \times mn) + (2 \times 5)\ (l^2m^2n^2 \times nl)$

$= (6)\ (l^{2+1}m^{2+1}n^2) - (4)\ (l^2m^{2+1}n^{2+1}) + (10)\ (l^{2+1}m^2n^{2+1})$

$= (6)\ (l^3m^3n^2) - (4)\ (l^2m^3n^3) + (10)\ (l^3m^2n^3)$

$= 6l^3m^3n^2 - 4l^2m^3n^3 + 10l^3m^2n^3$





## EXERCISE 8.3

1. Multiply

(i) $7m$ and $-8$ (ii) $2ab$ and $3a^2b^2$ (iii) $4xy$ and $2x^2y$

(iv) $-4ab$ and $-2bc$ (v) $3lm^3$ and $3mn$

(vi) $-6x^2y$ and $3xyz^2$ (vii) $2a^2b$ and $5a^2b^3$

(viii) $l^2mn$ and $lm^3n^6$ (ix) $-4x^2yz^7$ and $8xy^4z^3$

2. Simplify

(i) $lm\ (l + m)$ (ii) $2p(p + q)$ (iii) $3a\ (a - b)$

(iv) $2x\ (3x + 4y)$ (v) $2a\ (2b - 2c)$ (vi) $2lm\ (l^2m^2 - n)$

(vii) $a\ (a + b - c\ )$ (viii) $3x(\ x - 2\ y - 2z)$

(ix) $3p^2q\ (p^3 + q^2 - r^4)$

- **Multiplying binomial with Binomial / Trinomial**

**Example 3:** Multiply:

(i) $(x + 3)\ (x - 1)$ (ii) $(2a + 3b)\ (2a - 3b)$

(iii) $(m + 2)(m^2 - 2m + 3)$ (iv) $(2x - 1)\ (x^2 - 5x + 6)$

**Solution:**

(i) $(x + 3)\ (x - 1)$

$x + 3$

$\times\ \ x - 1$

$x^2 + 3x$

$-\ x - 3$

$x^2 + 2x - 3$

Thus, $(x + 3)\ (x - 1)$

$= x^2 + 2x - 3$

(ii) $(2a + 3b)\ (2a - 3b)$

$(2a + 3b)$

$\times\ \ (2a - 3b)$

$\because\ 2a \times 3b = 6ab$

$4a^2 + 6ab$

$-\ 6ab - 9b^2$

$4a^2 \qquad - 9b^2$

Thus, $(2a + 3b)\ (2a - 3b)$

$= 4a^2 - 9b^2$

(iii) $(m + 2)(m^2 - 2m + 3)$

$(m + 2)$

$\times\ \ (m^2 - 2m + 3)$

$m^3 - 2m^2 + 3m$

$+\ 2m^2 - 4m + 6$

$m^3 \qquad - m + 6$

Thus, $(m + 2)(m^2 - 2m + 3)$

$= m^3 - m + 6$

**Example 4:** Simplify:

(i) $2x^2\ (x^3 - x) - 3x\ (x^4 - 2x) + 2\ (x^4 - 3x^2)$

(ii) $(5a^2 - 6a + 9)\ (2a - 3) - (2a^2 - 5a + 4)\ (5a + 1)$

**Solution:**

(i) $2x^2\ (x^3 - x) - 3x\ (x^4 - 2x) + 2\ (x^4 - 3x^2)$

$= (2x^2 \times x^3 - 2x^2 \times x) - (3x \times x^4 - 3x \times 2x) + (2x^4 - 6x^2)$

$= (2x^{2+3} - 2x^{2+1}) - (3x^{1+4} - 6x^{1+1}) + (2x^4 - 6x^2)$

$= (2x^5 - 2x^3) - (3x^5 - 6x^2) + (2x^4 - 6x^2)$

$= 2x^5 - 2x^3 - 3x^5 + 6x^2 + 2x^4 - 6x^2$

$= (2x^5 - 3x^5) + 2x^4 - 2x^3 + (6x^2 - 6x^2)$

$= -x^5 + 2x^4 - 2x^3$

(ii) $(5a^2 - 6a + 9)(2a - 3) - (2a^2 - 5a + 4)(5a + 1)$

$$\begin{array}{r} 5a^2 - 6a + 9 \\ \times \quad 2a - 3 \\ \hline 10a^3 - 12a^2 + 18a \\ -15a^2 + 18a - 27 \\ \hline 10a^3 - 27a^2 + 36a - 27 \end{array} \qquad \begin{array}{r} 2a^2 - 5a + 4 \\ \times \quad 5a + 1 \\ \hline 10a^3 - 25a^2 + 20a \\ +2a^2 - 5a + 4 \\ \hline 10a^3 - 23a^2 + 15a + 4 \end{array}$$

$(5a^2 - 6a + 9)(2a - 3) - (2a^2 - 5a + 4)(5a + 1)$

$= (10a^3 - 27a^2 + 36a - 27) - (10a^3 - 23a^2 + 15a + 4)$

$= 10a^3 - 27a^2 + 36a - 27 - 10a^3 + 23a^2 - 15a - 4$

$= (10a^3 - 10a^3) + (-27a^2 + 23a^2) + (36a - 15a) + (-27 - 4)$

$= -4a^2 + 21a - 31$

## EXERCISE 8.4

1. Multiply

| | | | |
|---|---|---|---|
| (i) | $(3a + 4)(2a - 1)$ | (ii) | $(m + 2)(m - 2)$ |
| (iii) | $(x - 1)(x^2 + x + 1)$ | (iv) | $(p - q)(p^2 + pq + q^2)$ |
| (v) | $(x + y)(x^2 - xy + y^2)$ | (vi) | $(a + b)(a - b)$ |
| (vii) | $(l - m)(l^2 - 2lm + m^3)$ | (viii) | $(3p - 4q)(3p + 4q)$ |
| (ix) | $(1 - 2c)(1 + 2c)$ | (x) | $(2x - 1)(4x^2 + 2x + 1)$ |
| (xi) | $(a + b)(a^2 - ab + b^2)$ | (xii) | $(3 - b)(2b - b^2 + 3)$ |

2. Simplify

(i) $(x^2 + y^2)(3x + 2y) + xy(x - 3y)$

(ii) $(4x + 3y)(2x - y) - (3x - 2y)(x + y)$

(iii) $(2m^2 - 5m + 4)(m + 2) - (m2 + 7m - 8)(2m - 3)$

(iv) $(3x^2 + 2xy - 2y^2)(x + y) - (x^2 - xy + y^2)(x - y)$

## 8.3 Algebraic Identities

An algebraic identity is a simplified form consisting of the algebraic terms which provide us with a rule for solving the long calculations in a short and easy way. For example, to calculate the area of four rectangular walls we use the following identity as a short



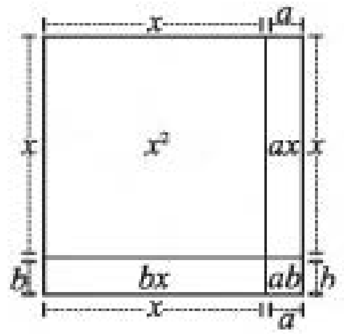


method.

Area of four walls = $2(l + b) \times h$

Now we learn some important algebraic identities.

**Identity 1:** $(x + a)(x + b) = x + (a + b)x + ab$




**Proof:**

L.H.S. $= (x + a)(x + b)$

$= x(x + b) + a(x + b)$

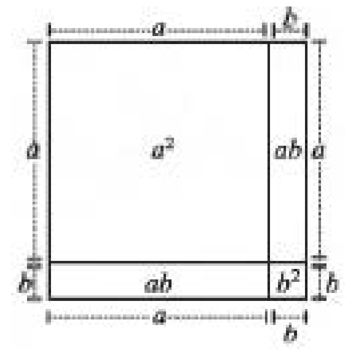
$= x^2 + bx + ax + ab$

$= x^2 + (b + a)x + ab$

$= x^2 + (a + b)x + ab$ = R.H.S.

Thus L.H.S. = R.H.S.

**Identity 2:** $(a + b)^2 = a^2 + 2ab + b^2$




**Proof:**

L.H.S. $= (a + b)^2 = (a + b)(a + b)$

$= a(a + b) + b(a + b)$

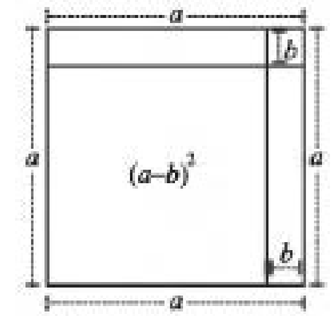
$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

Thus L.H.S. = R.H.S.

**Identity 3:** $(a - b)^2 = a^2 - 2ab + b^2$




**Proof:**

L.H.S. $= (a - b)^2 = (a - b)(a - b)$

$= a(a - b) - b(a - b)$

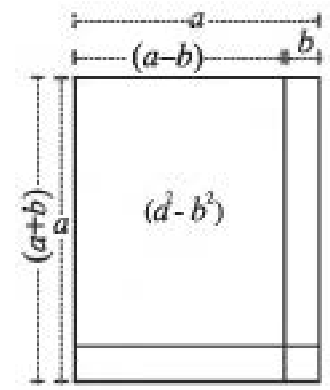
$= a^2 - ab - ba + b^2$

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

Thus L.H.S. = R.H.S.

**Identity 4:** $(a - b)(a + b) = a^2 - b^2$




**Proof:**

L.H.S. $= (a - b)(a + b)$

$= a(a + b) - b(a + b)$

$= a^2 + ab - ba - b^2$

$= a^2 + ab - ab - b^2$

$= a^2 - b^2$

Thus L.H.S. = R.H.S.

**Example 1:** Simplify the binomials by using the identity.

(i) $(x + 6)(x + 5)$ (ii) $(x - 4)(x - 8)$ (iii) $(2x + 9)(2x - 3)$

**Solution:**

(i) $(x + 6)(x + 5)$

By using the identity,

$(x + a)(x + b) = x + (a + b)x + ab$

$(x + 6)(x + 5) = x + (6 + 5)x + (6 \times 5)$

$= x^2 + 11x + 30$

(ii) $(x - 4)(x - 8)$

*By using the identity,*

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x - 4)(x - 8) = x^2 + (-4 - 8)x + (-4) \times (-8)$

$= x^2 - 12x + 32$

(iii) $(2x + 9)(2x - 3)$

By using the identity,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(2x + 9)(2x - 3) = (2x)^2 + (9 - 3)2x + 9 \times (-3)$

$= 4x^2 + (6)2x + (-27)$

$= 4x^2 + 12x - 27$

**Example 2:** Find the square of the following by using identity.

(i) $(4a + 3b)$ (ii) $(2x - 3y)$

**Solution:**

(i) $(4a + 3b)$

By using the identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$(4a + 3b)^2 = (4a)^2 + 2 \times (4a) \times (3b) + (3b)^2$

$= 16a^2 + 24ab + 9b^2$

(ii) $(2x - 3y)$

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$(2x - 3y)^2 = (2x)^2 + 2 \times (2x) \times (3y) + (3y)^2$

$= 4x^2 - 4xy + 9y^2$

**Example 3:** Write the product of the following binomials by using identity,

(i) $(3x - 4y),(3x + 4y)$ (ii) $(7a - 9b), (7a + 9b)$

(iii) $(6x^2y^2 + 8a^2b^2),(6x^2y^2 - 8a^2b^2)$





**Solution:**

(i) $(3x - 4y),(3x + 4y)$

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(3x + 4y)(3x - 4y) = (3x)^2 - (4y)^2$

$= 9x^2 - 16y^2$

(ii) $(7a - 9b), (7a + 9b)$

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(7a + 9b)(7a - 9b) = (7a)^2 - (9b)^2$

$= 49a^2 - 81b^2$

(iii) $(6x^2y^2 + 8a^2b^2), (6x^2y^2 - 8a^2b^2)$

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(6x^2y^2 + 8a^2b^2)(6x^2y^2 - 8a^2b^2) = (6x^2y^2)^2 - (8a^2b^2)^2$

$= 36x^4y^4 - 64a^4b^4$

## EXERCISE 8.5

1. Simplify the following binomials by using the identity.
   (i) $(x + 1)(x + 2)$ (ii) $(x - 2)(x - 4)$ (iii) $(a + 5)(a + 3)$
   (iv) $(b + 6)(b - 9)$ (v) $(2x + 3)(2x - 7)$ (vi) $(2y + 1)(2y + 5)$
   (vii) $(3b - 1)(3b - 7)$ (viii) $(4x + 5)(4x + 3)$ (ix) $(5y - 2)(5y + 6)$
   (x) $(8a + 7)(8a - 3)$
2. By using identity, find the square of the following binomials.
   (i) $x + y$ (ii) $3a + 4$ (iii) $x - y$ (iv) $a + 2b$ (v) $2x + 3y$
   (vi) $2a - b$ (vii) $3x - 2y$ (viii) $4x + 5y$ (ix) $7a - 8b$
3. Find the product of the following binomials by using identity.
   (i) $(x + y)(x - y)$ (ii) $(3a - 8)(3a + 8)$ (iii) $(2a + 7b)(2a - 7b)$
   (iv) $(x + 3y)(x - 3y)$ (v) $(6a - 5b)(6a + 5b)$ (vi) $(9x - 11y)(9x + 11y)$

## 8.4 Factorization of Algebraic Expressions

In arithmetic, we have learnt that the prime numbers which are multiplied with each other to get a product are called factors. For example, 18 = 1 x 2 x 3 x 3 .................... (i)

Similarly in algebra, if an algebraic expression is a product of two or more than two other algebraic expressions, then the two or more than two other algebraic expressions are called the factors of the product. For example,

$3xy - 3xz = 3x(y - z)$ ............. (ii)

Here in (ii), 3, $x$ and $(y - z)$ are the factors of $3xy - 3xz$ and 3 & $x$ are known as common factors of the whole expression. So, we can define the factorization of an algebraic expression as,

"The process of writing an algebraic expression as the product of two or more expressions which divide it exactly is called the factorization".

In algebra, the opposite of the factorization is called the expansion. This is the process of multiplying the factors to get the same algebraic expression.

**Example 1:** Resolve the following expressions into factors.

(i) $3a + 6b + 9c$ (ii) $a(x - y) - b(x - y)$

**Solution:**

(i) $3a + 6b + 9c$

(taking 3 as common)

$= 3(a + 2b + 3c)$

(ii) $a(x - y) - b(x - y)$

(taking $x - y$ as common)

$= (x - y)(a - b)$

**Example 2:** Factorize.

(i) $(ax - y) - (ay - x)$ (ii) $(x^2 + yz) - (y + z)x$

**Solution:**

(i) $(ax - y) - (ay - x)$

$= ax - y - ay + x$

$= ax + x - ay - y$

$= x(a + 1) - y(a + 1)$

$= (x - y)(a + 1)$

(ii) $(x^2 + yz) - (y + z)x$

$= x^2 + yz - yx - zx$

$= x^2 - zx - yx + yz$

$= x(x - z) - y(x - z)$

$= (x - y)(x - z)$

- **Factorization of $a^2 - b^2$ type expression**

If we have the difference of two squared terms, then we can factorize them as one factor is the sum of two terms and the other factor is the difference of two terms. For example, the difference of





two squared terms is $a^2 - b^2$. So,

$a^2 - b^2 = a^2 + ab - ab - b^2$

$= a(a + b) - b(a + b)$

$= (a - b)(a + b)$

**Example 1:** Factorize.

(i) $49x^2 - 81y^2$ (ii) $18a^2x^2 - 32b^2y^2$ (iii) $(6a - 8b)^2 - 49c^2$

**Solution:**

(i) $49x^2 - 81y^2$

$= (7x)^2 - (9y)^2$

$= (7x - 9y)(7x + 9y)$

[Using the identity, $a^2 - b^2 = (a - b)(a + b)$]

(ii) $18a^2x^2 - 32b^2y^2$

$= 2[9a^2x^2 - 16b^2y^2]$

$= 2[(3ax)^2 - (4by)^2]$

$= 2(3ax - 4by)(3ax + 4by)$

[Using the identity, $a^2 - b^2 = (a - b)(a + b)$]

(iii) $(6a - 8b)^2 - 49c^2$

$= (6a - 8b)^2 - (7c)^2$

$= (6a - 8b - 7c)(6a - 8b + 7c)$

[Using the identity, $a^2 - b^2 = (a - b)(a + b)$]

## EXERCISE 8.6

1. Resolve into factors.

(i) $5x^2y - 10xy^2$ (ii) $2a - 4b + 6c$ (iii) $9x^4 + 6y^2 + 3$
(iv) $a^3b + a^2b^2 + ab^3$ (v) $x^2yz + xy^2z + xyz^2$ (vi) $bx^3 + bx^2 - x - 1$
(vii) $x^2 + qx + px + pq$ (viii) $ab - a - b + 1$ (ix) $(pm + n) + (pn + m)$
(x) $(a^2 + bc) - (b + c)a$ (xi) $x^2 - (m + n)x + mn$ (xii) $x^3 - y^2 + x - x^2y^2$

2. Factorize by using identity.

(i) $4a^2 - 25$ (ii) $4x^2 - 9y^2$ (iii) $9a^2 - b^2$
(iv) $9m^2 - 16n^2$ (v) $16b^2 - a^2$ (vi) $-1 + (x + 1)^2$
(vii) $8x^2 - 18y^2$ (viii) $(a + b)^2 - c^2$ (ix) $x^2 - (y + z)^2$ (x) $7x^2 - 7y^2$
(xi) $5a^2 - 20b^2$ (xii) $x^4 - y^4$

- **Factorization of $a^2 \pm 2ab + b^2$ type expressions**

We know that the square of a binomial can be expanded as the square of 1st term plus the square of 2nd term plus the twice of the product of the two terms, i.e.

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

**Example 3:** Resolve into factors.

(i) $8x^2 - 56x + 98$ (ii) $16a^4 + 14a^2b^2 + 9b^2$

**Solution:**

(i) $8x^2 - 56x + 98 = 2[4x^2 - 28x + 49]$

It can be written as:

$= 2[(2x)^2 - 2(2x)(7) + (7)^2]$ $\because 28x = 2\,(2x)\,(7)$

$= 2(2x - 7)^2$

[Using the identity, $a^2 - 2ab + b^2 = (a - b)^2$]

Thus, the required factors are 2 and $(2x - 7)^2$.

(ii) $16a^4 + 14a^2b^2 + 9b^2$

$= (4a^2)^2 + 2(4a^2)\,(3b^2) + (3b^2)^2$ $\because 2(4a^2)\,(3b^2) = 24a^2b^2$

By using the identity, $a^2 + 2ab + b^2 = (a + b)^2$, we have

$(4a^2)^2 + 2(4a^2)\,(3b^2) + (3b^2)^2 = (4a^2 + 3b^2)^2$

Thus, the required factors are $(4a^2 + 3b^2)^2$

**Example 2:** Factorize $\frac{l^2}{m^2}a^2 - \frac{2l}{n}ab + \frac{m^2}{n^2}b^2$

**Solution:** $\frac{l^2}{m^2}a^2 - \frac{2l}{n}ab + \frac{m^2}{n^2}b^2$

It can be written as,

$= \left(\frac{l}{m}a\right)^2 - 2\left(\frac{l}{m}a\right)\left(\frac{m}{n}b\right) + \left(\frac{m}{n}b\right)^2$ $\because 2\left(\frac{l}{m}a\right)\left(\frac{m}{n}b\right) = 2\frac{l}{n}ab$

By using the identity, $a^2 - 2ab + b^2 = (a - b)^2$

$$\left(\frac{l}{m}a\right)^2 - 2\left(\frac{l}{m}a\right)\left(\frac{m}{n}b\right) + \left(\frac{m}{n}b\right)^2 = \left(\frac{l}{m}a - \frac{m}{n}b\right)^2$$

Thus, the required factors are $\left(\frac{l}{m}a - \frac{m}{n}b\right)^2$





## EXERCISE 8.7

1. Resolve into factors by using identity.

(i) $x^2 + 8x + 16$ (ii) $x^2 - 2x + 1$
(iii) $a^4 - 14a^2 + 49$ (iv) $1 + 10m + 25m^2$
(v) $4x^2 - 12xy + 9y^2$ (vi) $9a^2 + 30ab + 25b^2$
(vii) $16a^2 + 56ab + 49b^2$ (viii) $36x^2 + 108xy + 81y^2$
(ix) $49m^2 + 154m + 121$ (x) $64a^2 - 208ab + 169b^2$
(xi) $3x^4 + 24x^2 + 48$ (xii) $11x^2 + 22x + 11$
(xiii) $44a^4 - 44a^3b + 11a^2b^2$ (xiv) $a^4 + 16a^2b + 64b^2$
(xv) $1 - 4xyz + 4x^2y^2z^2$ (xvi) $16x^3y - 40x^2y^2 + 25xy^3$

2. Factorize by using the identity.

(i) $a^2x^2 + 2abcx + b^2c^2$ (ii) $\frac{l^2}{4} + lmn + m^2n^2$
(iii) $\frac{4}{9}x^2 - xy + \frac{9}{16}y^2$ (iv) $\frac{121}{169}a^2 - 2ab + \frac{169}{121}b^2$
(v) $\frac{a^2x^2}{b^2} - \frac{2axy}{c} + \frac{b^2y^2}{c^2}$ (vi) $\frac{l^4}{n}x^4 - 2\frac{l^2m^2}{n}x^2y^2 + \frac{m^4}{n}y^4$
(vii) $a^2b^2c^2x^2 - 2a^2b^2cdxy + a^2b^2d^2y^2$ (viii) $\frac{b^2}{c^2}x^4 + \frac{2b}{a}x^3y + \frac{c^2}{a^2}x^2y^2$

- **Factorization by making groups**

Look at the following algebraic expressions.

- $x^2 + ax + 4x + 4a$
- $al + bm + bl + am$
- $pq - 2p - q + 2$

We can observe from the above given expressions. That there are no common factors in them and these expressions are also not any of the three types discussed in other sections. For factoring such types of expressions, we rearrange them and make their groups as given in the examples.

**Example 1:** Factorize $5a + xa + 5x + x^2$

**Solution:**

$5a + xa + 5x + x^2$

**Step 1:** Rearrange the expression. $x^2 + 5x + xa + 5a$

**Step 2:** Make their groups. $(x^2 + 5x) + (xa + 5a)$

**Step 3:** Factor out the common factors. $x(x + 5) + a(x + 5)$
**Step 4:** Factor out the common expression. $(x + 5)(x + a)$
Thus, the required factorization is $(x + 5)(x + a)$

**Example 2:** Factorize: $2a^2b + 4ab^2 - 2ab - 4b^2$

**Solution:**

$2a^2b + 4ab^2 - 2ab - 4b^2$

**Step 1:** Rearrange the expression and factor out the common factor. $2a^2b - 2ab + 4ab^2 - 4b^2$
$= 2b(a^2 - a + 2ab - 2b)$
**Step 2:** Make their groups. $= 2b[(a^2 - a) + (2ab - 2b)]$
**Step 3:** Again factor out the common factors. $= 2b[a(a - 1) + 2b(a - 1)]$
**Step 4:** Factor out the common expression. $= 2b[(a - 1)(a + 2b)]$
Thus, the required factorization is $2b(a - 1)(a + 2b)$.

## EXERCISE 8.8

1. Factorize the following expressions.
(i) $lx - my + mx - ly$ (ii) $2xy - 6yz + x - 3z$ (iii) $p^2 + 2p - 3p - 6$
(iv) $x^2 + 5x - 2x - 10$ (v) $m^2 - 7m + 2m - 14$ (vi) $a^2 + 3a - 4a + 12$
(vii) $x^2 - 9x + 3x - 27$ (viii) $z^2 - 8z - 4z + 32$ (ix) $t^2 - st + t - s$
(x) $n^2 + 5n - n - 5$ (xi) $a^2b^2 + 7ab - ab - 7$ (xii) $l^2m^2 - 13lm - 2lm + 26$

## REVIEW EXERCISE 8

1. Answer the following questions.
(i) What is meant by literals? (ii) Define a constant.
(iii) What is a binomial?
(iv) What is an algebraic identity?
(v) Define the factorization of an algebraic expression.
2. Fill in the blanks.
(i) $(a + b)^2 =$ ______. (ii) $(a - b)^2 =$ ______.
(iii) $(x + a)(x + b) =$ ______. (iv) $a^2 - b^2 =$ ______.
(v) A symbol represented by a literal and can take various numerical values is called a ______.
(vi) A polynomial having only one term is called ______.





3. Tick (✓) the correct answer.

4. Resolve into factors.
(i) $10a^2 - 200a^4b$
(ii) $36x^3y^3z^3 - 27x^2y^4z + 63xyz^4$
(iii) $15x^4y + 21x^3y^2 - 27x^2y^2 - 33xy^4$
(iv) $x(a^2 + 11) - 16(a^2 + 11)$
(v) $x^2(ab + c) + xy(ab + c) + z^2(ab + c)$
5. If $A = 2(x^2 + y^2 + z^2)$, $B = -x^2 + 3y^2 - 2z^2$ and $C = x^2 - y^2 - 3z^2$, then find:
(i) $A + B + C$ (ii) $B + C - A$ (iii) $A - B + C$
(iv) $A + B - C$ (v) $A - B - C$ (vi) $B - C - A$
6. Simplify the following polynomials
(i) $(x - 2y)(x + 2y)$ (ii) $(4x^2)(3x + 1)$
(iii) $2x(x + y) - 2y(x - y)$ (iv) $(a^2b^3)(2a - 3b)$
(v) $(a^2 - b^2)(a^2 + b^2)$ (vi) $(a^2 + 1)(a^2 - a - 1)$
(vii) $x(y + 1) - y(x + 1) - (x - y)$
(viii) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$
7. Simplify the following by using identity.
(i) $(3x - 4)(3x + 5)$ (ii) $(2a - 5b)^2$
8. Factorize.
(i) $a^2 - 26a + 169$ (ii) $1 - 6x^2y^2z + 9x^4y^4z^2$
(iii) $7ab^2 - 343a$ (iv) $75 - 3(x - y)^2$
(v) $49(x + y)^2 - 16(x - y)^2$ (vi) $\frac{9}{16}a^2 + ab + \frac{4}{9}b^2$
(vii) $\frac{a^2}{b^2}l^2 - \frac{2ac}{bd}lm + \frac{c^2}{d^2}m^2$ (viii) $(a - \frac{9}{5})^2 - \frac{36}{25}m^2$

## SUMMARY

- The letters or alphabets that we use to represent unknowns / numbers are called literals.
- A symbol represented by a literal that can take various numerical values is called a variable.
- A symbol having a fixed value is called a constant.
-
- A combination of constants and variables connected by the signs of fundamental operations is called an algebraic expression.
- The parts of an algebraic expression separated by the operational signs '+' and '–' are called its terms.
- An algebraic expression in which the exponents of variables are all whole numbers is called a polynomial.
- A polynomial can be arranged in any order but usually we arrange it in descending order.
- An algebraic equation which is true for all values of the variable occurring in the relation is called an algebraic identity.
- The process of writing an algebraic expression as the product of two or more expressions which divide it exactly is called the factorization.

CHAPTER

# 9

# LINEAR EQUATIONS

*Animation 9.1: Linear Equation*
*Source & Credit: eLearn.Punjab*

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Define a linear equation in one variable.
- Demonstrate different techniques to solve linear equations.
- Solve linear equations of the type:
  - $ax+b=c,$
  - $\frac{ax+b}{cx+d}=\frac{m}{n}.$
- Solve real life problems involving linear equations.

## 9.1 Linear Equation

The equation which contains a single variable with the exponent of 1 is called the linear equation in one variable. For example,

- $2x + 4 = 6x$ (Linear equation in variable $x$)
- $3y - 7 = 14 - 2y$ (Linear equation in variable $y$)
- $z + 5 = 0$ (Linear equation in variable $z$)

## 9.2 Solution of a Linear Equation

A linear equation in one variable is an open sentence. The process of finding that value of the variable which makes it a true sentence is called its solution. That value of the variable which makes the equation a true sentence is called a solution of the equation. A solution is also called a root of the equation.

(i) $x + 2 = 5$

Here solution is $x = 3$ or the root is $x = 3$ because when we put $x = 3$, we get 5 = 5 which is a true statement.

(ii) $2x = 4$

We put $x = 2$ and get 4 = 4, a true statement, thus the solution of the equation is $x = 2$.





| Equation | Left-hand side | Right-hand side |
|---|---|---|
| $x + 3 = 6$ | $x + 3$ | 6 |
| $2x - 5 = 5$ | $2x - 5$ | 5 |
| $6 = 12 + x$ | 6 | $12 + x$ |

- **Addition**

We can add the same number to both sides of an equation. For example, if we are given an equation.

$x + 2 = 4$ ... (i)

We can add 3 to both sides of (i) to obtain:

$x + 2 + 3 = 4 + 3$

or $x + 5 = 7$ ... (ii)

(i) and (ii) are equivalent equations which have the same solution or root

- **Subtraction**

We can subtract the same number from the both sides of an equation. For example;

$x + 5 = 3$ ... (i)

$x + 5 - 2 = 3 - 2$

or $x + 3 = 1$ ... (ii)

(i) and (ii) are equivalent equations.

- **Multiplication**

We can multiply both sides of an equation by a non-zero number. For example:

$\frac{1}{4}x = 8$ ... (i)

Multiply both sides by 4

$4 \times \frac{1}{4}x = 8 \times 4$

or $x = 32$ ... (ii)

- **Division**

We can divide both sides of an equation by an non-zero number. For example:

$6x = 12$ ... (i)

Multiply both sides by 4

$4 \times - = 8 \times 4$

$x = 32$ ... (ii)

**Example 1:** Solve the equation, $x - 6 = 2$.

**Solution:**

$x - 6 = 2$ .......... (i)

Add 6 to both sides,

$x - 6 + 6 = 2 + 6$

$x = 8$

**Example 3:** Solve the equation, $x + 1 = 5$.

**Solution:**

$x + 1 = 5$ .......... (i)

Subtract 1 from both sides of (i),

$x + 1 - 1 = 5 - 1$

$x = 4$

**Example 2:** Solve the equation,

$\frac{1}{6}x = 2$.

**Solution:**

$\frac{1}{6}x = 2$

Multiply both sides of (i) by 4

$6 \times \frac{1}{6}x = 6 \times 2$

or $x = 12$

**Example 4:** Find the solution of the following equations and verify the solution.

(i) $\frac{x+6}{2} = \frac{x+4}{3}$ (ii) $\frac{8x+4}{16-4x} = 1$

**Solution:**

(i) $\frac{x+6}{2} = \frac{x+4}{3}$

$6 \times \frac{x+6}{2} = 6 \times \frac{x+4}{3}$ (Multiply both sides by the L.C.M 6 of 2 and 3)

$3(x + 6) = 3(x + 6)$

$3x + 18 = 2x + 8$

$3x - 2x = 8 - 18$

$x = -10$ (Separate variables and numbers)





(ii) $\frac{8x+4}{16-4x} = 1$

or $(16-4x) \times \frac{8x+4}{16-4x} = 1 \times (16-4x)$

or $8x + 4 = 16 - 4x$ (Multiply both sides by the L.C.M $16 - 4x$)

or $8x + 4x = 16 - 4$ (Separate variables and numbers)

or $12x = 12$

or $x = \frac{12}{12} = 1$

## EXERCISE 9.1

1. Solve the following equations.

| | | |
|---|---|---|
| (i) $\frac{1}{8}x = 4$ | (ii) $x - 7 = -15$ | (iii) $x + 1 = 5$ |
| (iv) $2x - 6 = 0$ | (v) $11x - 2 = 20$ | (vi) $17x = 255$ |
| (vii) $5x - 3 = 12$ | (viii) $11 - x = 6$ | (ix) $\frac{2x}{5} = 8$ |
| (x) $\frac{x}{3} - 7 = 2$ | (xi) $\frac{5x}{2} = 10$ | (xii) $9x + 11 = 83$ |
| (xiii) $\frac{x-5}{4} = 7$ | (xiv) $\frac{x}{4} - 2 = 5$ | (xv) $\frac{7x+3}{2} = 19$ |

2. Find the solutions of the following equations.

| | | |
|---|---|---|
| (i) $5x-3 = 3x-5$ | (ii) $3x+8 = 5x+2$ | (iii) $12x-3 = 5(2x+1)$ |
| (iv) $10(2-x) = 4(x-9)$ | (v) $\frac{x-3}{x+1} = \frac{3}{5}$ | (vi) $\frac{x-1}{x-2} = \frac{4}{3}$ |
| (vii) $\frac{x-2}{3x+4} = \frac{1}{7}$ | (viii) $\frac{3x-8}{5x-2} = 1$ | (ix) $\frac{x+2}{2x-5} = \frac{2}{5}$ |
| (x) $\frac{x+3}{2} = \frac{x+6}{3}$ | (xi) $\frac{7x-6}{x-18} = 1$ | (xii) $\frac{4x+3}{3} = \frac{x+7}{2}$ |

### 9.2.1 Solving Real Life Problems involving Linear Equations

Let us solve some real life problems involving linear equations.

**Example 1:** A 96cm long wire is given the shape of a rectangle such that its length is 12cm more than the breadth. Find the length and breadth of the rectangle.

**Solution:**

Suppose that breadth of the rectangle = $x$
then length of the rectangle = $x + 12$
length of the wire (perimeter) = 96cm
By using the formula

2(length + breadth) = perimeter

or $2[(x + 12) + x)] = 96$
or $2(2x+ 12) = 96$
or $4x + 24 = 96$
or $4x = 96 - 24$
or $4x = 72$
or $x = 18$

Thus, breadth of the rectangle is 18cm

Length of the rectangle = $x + 12$
$= 18 + 12 = 30$cm

**Example 2:** After 32 years from now, a boy will be 5 times as old as he was 8 years back. How old is the boy now?

**Solution:**

Suppose the age of the boy = $x$
After 32 years age will be = $x + 32$
8 years back the age was = $x - 8$
According to the situation,

$x + 32 = 5 (x - 8)$
or $x + 32 = 5x - 40$
or $5x - x = 40 + 32$
or $4x = 72$
or $x = 72/4 = 18$ years





Thus, the boy is 18 years old.

## EXERCISE 9.2

1. Hussain bought 10 ice creams. He gave Rs. 1,000 to the shopkeeper. The shopkeeper returned him Rs. 250. For how much did he buy one ice cream?
2. The length of a rectangle is 2 cm more than twice its breadth. If the perimeter of the rectangle is 28cm, find its length and breadth.
3. The price of a pen is Rs. 42 and of a notebook is Rs. 18. Calculate how many pens and notebooks you can buy for Rs. 480 if you want to buy an equal quantity of both.
4. A father's age is twice his daughter's age but 16 years ago the father's age was 4 times his daughter's age. Calculate their ages.
5. Distribute an amount of Rs. 200 between Raheem and Usman such that Raheem gets Rs.50 more than twice as much as Usman gets.
6. The length of a marriage hall is 4 times its breadth. If the perimeter of the hall is 240m, find the length and the breadth of the marriage hall.
7. Aslam's age is half of his father's age but 15 years ago his age was just $\frac{1}{3}$ of father's age. Find his present age now.
8. Distribute an amount of Rs.500 among 2 brothers and 1 sister such that,
   a. sister gets twice as much as brothers gets.
   b. each brother gets twice as much as the sister does.

## REVIEW EXERCISE 9

1. Answer the following questions.
   (i) What is a linear equation?
   (ii) What is meant by the solution of an equation?
   (iii) Define the linear equation in one variable.

2. Fill in the blanks.
   (i) The equation which contains a single variable with the exponent 1 is called the linear equation in one______.
   (ii) A solution is also called a _____ of the equation.
   (iii) The process of finding the value of a variable to make a sentence true is called its______.
   (iv) Addition of the ______ to both sides of an equation does not affect its equality.
3. Tick (✓) the correct answer.

4. Solve each of the following equations.

| | | | |
|---|---|---|---|
| (i) | $2x + 3 = 5x + 7$ | (ii) | $5x - \frac{5}{3} = 3x - \frac{2}{3}$ |
| (iii) | $\frac{3}{2}x - \frac{5}{3} = \frac{5}{2} + \frac{7}{3}x$ | (iv) | $3(3x-1) - 8(x + \frac{3}{2}) = 0$ |
| (v) | $\frac{5}{2}(\frac{3}{2} - 2x) + \frac{3}{2}(2x - \frac{5}{2}) = 0$ | (vi) | $\frac{2}{3} - \frac{2}{3}x = \frac{3}{2}x - \frac{1}{3}$ |
| (vii) | $2 - \frac{3}{2}x = \frac{5}{2}(1-x)$ | (viii) | $\frac{2}{5}(3x-1) = 2x - 1$ |
| (ix) | $\frac{1}{3}(x-3) + \frac{2}{3} = \frac{4x-3}{6}$ | (x) | $\frac{1}{3}(x-3) + \frac{2}{3} = \frac{1}{3}(4x-3) + \frac{7}{2}$ |

5. Find the number.
   (i) –3 added to a number is equal to 10.
   (ii) Three times a number is 15.
   (iii) 13 subtracted from three times a number is 8.
   (iv) A number divided by 5 gives 9 less than twice the number.
   (v) The sum of three consecutive numbers is 45.





## SUMMARY

- An equation which contains a single variable with the exponent "1" is called the linear equation in one variable.
- The value of the variable that makes the equation a true sentence is called the solution of the equation.
- A number non-zero in case of division can be added, subtracted, multiplied and divided on the both sides of an equation and it does not affect the equality of the equation.

CHAPTER

10

# FUNDAMENTALS OF GEOMETRY

*Animation 10.1: Circle radians*
*Source & Credit: .wikipedia*

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Define adjacent, complementary and supplementary angles.
- Define vertically opposite angles.
- Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
- Calculate unknown angle of a triangle.
- Identify congruent and similar figures.
- Recognize the symbol of congruency.
- Apply the properties for two figures to be congruent or similar.
- Apply following properties for congruency between two triangles.
  1. SSS $\cong$ SSS
  2. SAS $\cong$ SAS
  3. ASA $\cong$ ASA
  4. RHS $\cong$ RHS
- Describe a circle and its center, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
- Draw a semicircle and demonstrate the property; the angle in a semicircle is a right angle.
- Draw a segment of a circle and demonstrate the property; the angles in the same segment of a circle are equal.

# Introduction

Geometry has a long and glorious history. It helped us to create art, build civilizations, construct buildings and discover other worlds. Therefore, the knowledge of geometry remained the focus of ancient mathematicians.

The most important work done in the area of Geometry was of Euclid. His book,"Euclid's Elements" had been taught throughout the world.





## 10.1 Properties of Angles

Two different rays with a common starting point form an angle which is denoted by the symbol $\angle$. The unit of measuring an angle is degree (°). Angles are classified by their degree measures, e.g. right angle, acute angle, obtuse angle, etc

### 10.1.1 Adjacent, Complementary and Supplementary Angles

- **Adjacent Angles**

The word adjacent means "next or neighbouring" and by the adjacent angles we mean angles next to each other. Two angles are said to be adjacent if:

(i) they have the same vertex.
(ii) they have one common arm.
(iii) other arms of two angles extend on opposite sides of the common arm.

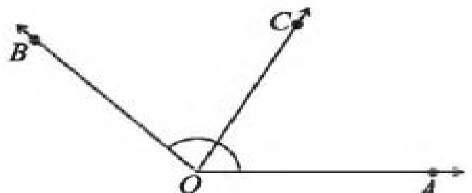


Figure 10.1

For example, in the figure (10.1), it can be seen that the two angles $\angle AOC$ and $\angle BOC$ are adjacent because they have the same vertex "$O$" and one common arm $\overrightarrow{OC}$. Other arms $\overrightarrow{OA}$ and $\overrightarrow{OB}$ on the opposite sides of the common arm $\overrightarrow{OC}$.

- **Complementary Angles**

Two angles are called complementary angles when their sum of degree measure is equal to 90°. For example, in the figure (10.2),

$m\angle BAD = 60^0$
$m\angle SAD = 30^0$
$m\angle BAD + m\angle SAD$
$= 60^0 + 30^0$
$= 90^0$

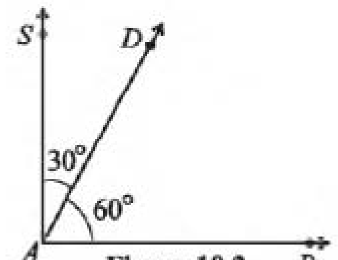


Figure 10.2

Thus $\angle BAD$ and $\angle SAD$ are complementary angles.

- **Supplementary Angles**

Two angles are called supplementary angles when their sum of degree measure is equal to 180°. For example, in the figure (10.3),

$m\angle BAD = 60^0$
$m\angle SAD = 120^0$
$m\angle BAD + m\angle SAD$
$= 60^0 + 120^0$
$= 180^0$

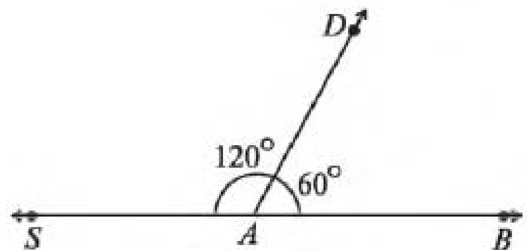


Figure 10.3

The sum of the two angles is 180°. Hence, these are supplementary angles.

## 10.1.2 Vertically Opposite Angles

A pair of angles is said to be vertically opposite, if the angles are formed from two intersecting lines and the angles are non-adjacent. Such angles are always equal in measurement as shown in the figure (10.4).

Here it can be seen that two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect each other at point "*O*" and form four angles i.e. $\angle AOC$, $\angle BOC$, $\angle BOD$ and $\angle AOD$. Here the angles $\angle AOD$ and $\angle BOC$ are called vertically opposite angles. Similarly the angles $\angle AOC$ and $\angle BOD$ are also vertically opposite to each other. We can prove that vertically opposite angles are equal in measure as given below. In the figure (10.4), we can also see that:

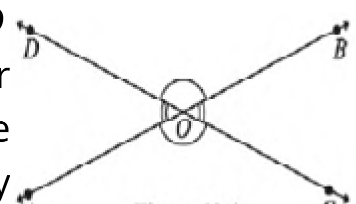


Figure 10.4

$m\angle AOD + m\angle AOC = 180^0$ (supplementary angles)
$m\angle AOC + m\angle BOC = 180^0$ (supplementary angles)
$m\angle AOD + m\angle AOC = m\angle AOC + m\angle BOC$
$m\angle AOD = m\angle BOC$

Similarly, we can also prove that:

$m\angle AOC = m\angle BOD$

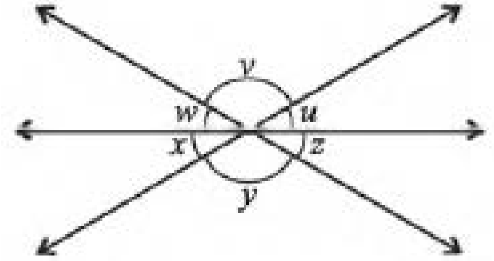




## 10.1.3 Finding Unknown Angles involving Adjacent, Complementary, Supplementary and Vertically Opposite Angles

**Example 1:** Look at the following diagram and name all the pairs of:

(a) Adjacent angles
(b) Vertically Opposite Angles




**Solution:**

**(a) Adjacent angles**

| | | | | | |
|---|---|---|---|---|---|
| (i) | $\angle u$ and $\angle v$ | (ii) | $\angle v$ and $\angle w$ | (iii) | $\angle w$ and $\angle x$ |
| (iv) | $\angle x$ and $\angle y$ | (v) | $\angle y$ and $\angle z$ | (vi) | $\angle z$ and $\angle u$ |

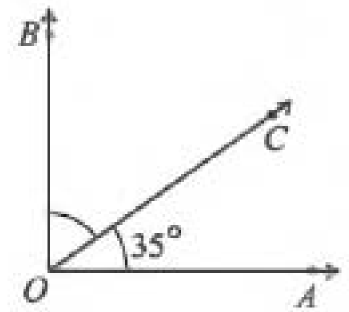
All these are pairs of adjacent angles

**(b) Vertically Opposite Angles**

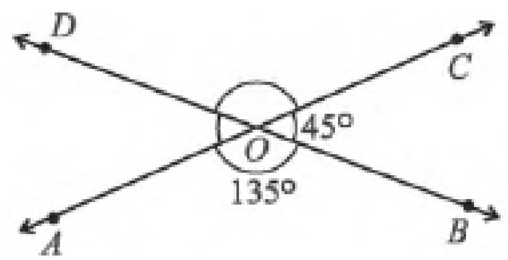
| | | | | | |
|---|---|---|---|---|---|
| (i) | $\angle u$ and $\angle x$ | (ii) | $\angle w$ and $\angle z$ | (iii) | $\angle v$ and $\angle y$ |

All these are pairs of vertically opposite angles.

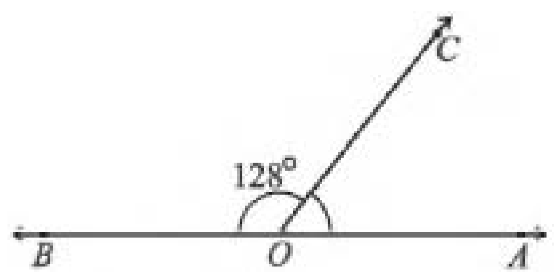
**Example 2:** Write the measurement of missing angles.



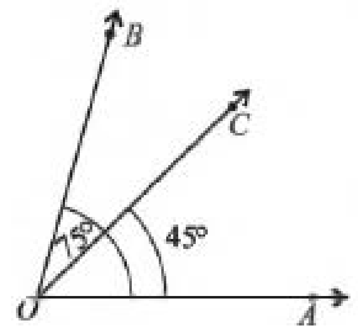
(i)



(ii)



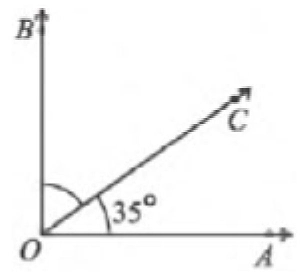
(iii)



(iv)

**Solution:**

(i)

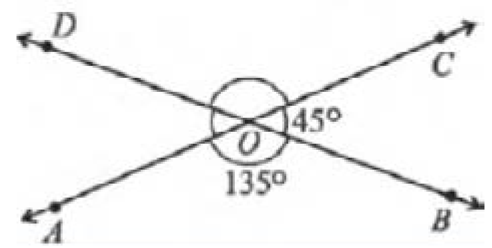



We have $m\angle AOC = 35°$.
Since, $\angle AOC$ and $\angle BOC$ are complementary angles. So,

$m\angle AOC + m\angle BOC = 90^0$
$35^0 + m\angle BOC = 90^0$
$m\angle BOC = 90^0 - 35^0$
$= 55^0$

Thus, $m\angle BOC = 55^0$

(ii)




We have to $\angle AOB = 135^0$ and $m\angle BOC = 45^0$. Since, $\angle AOB$ and $\angle COD$ are vertically opposite angles. So,

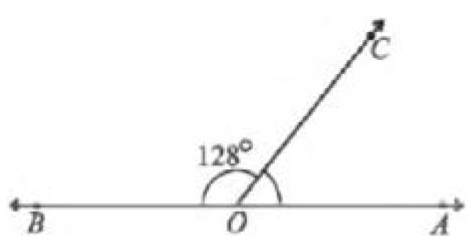
$m\angle COD = m\angle AOB$

Thus, $m\angle COD = 135^0$
Similarly, $\angle BOC$ and $\angle AOD$ are vertically opposite angles. So,

$m\angle AOD = m\angle BOC$

Thus, $m\angle AOD = 45^0$

(iii)

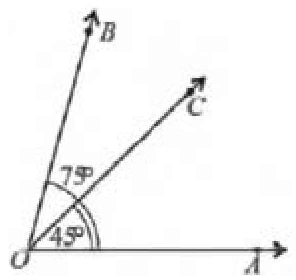



We have $m\angle BOC = 128^0$. Since, $\angle AOC$ and $\angle COB$ are supplementary angles. So,

$m\angle AOC + m\angle BOC = 180^0$
$m\angle AOC + 128^0 = 180^0$
$m\angle AOC = 180^0 - 128^0$
$= 52^0$

Thus, $m\angle AOC = 52^0$

(iv)




We have $m\angle AOB = 75^0$ and $\angle AOC = 45^0$. Since, $\angle AOC$ and $\angle COB$ are adjacent angles. So,

$m\angle AOC + m\angle BOC = \angle AOB$
$45^0 + m\angle BOC = 75^0$
$m\angle BOC = 75^0 - 45^0$
$= 30^0$

Thus, $m\angle BOC = 30^0$

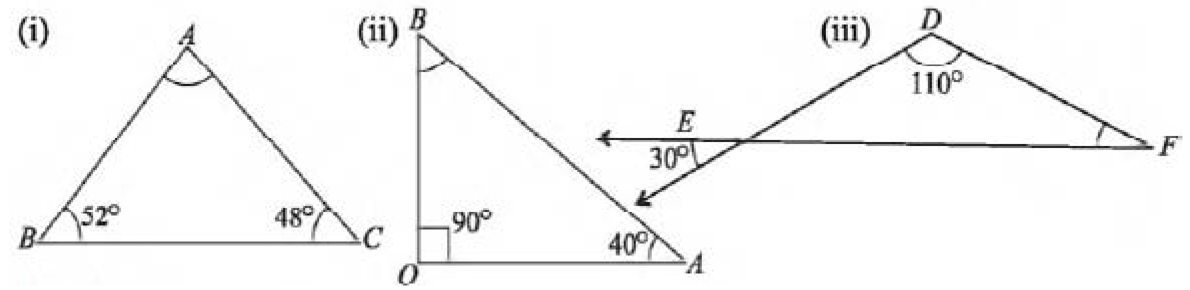
## 10.1.4 Finding Unknown Angle of a Triangle

If the measurements of two angles of a triangle are known, then the third angle can be calculated.



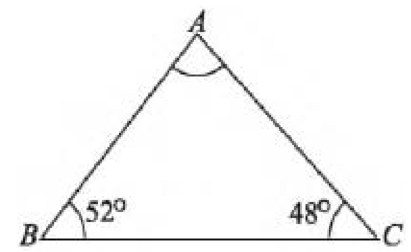


**Example 3:** Find the missing angle in each triangle.




**Solution:**
We know that the sum of the measures of three angles of a triangle is always equal to 180°. Let us use the same angle sum property of a triangle to find the following unknown angles.

(i)



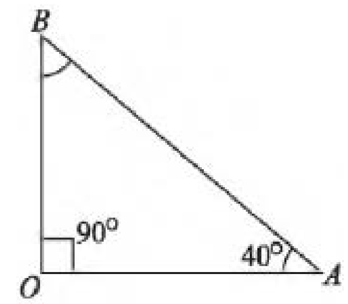

We have,
$m\angle B = 52^0$, $m\angle C = 48^0$, $m\angle A = ?$

We know that

$m\angle A + m\angle B + m\angle C = 180^0$
$m\angle A + 52^0 + 48^0 = 180^0$
$m\angle A + 100^0 = 180^0$
$m\angle A = 180^0 - 100^0 = 80^0$

Thus, $m\angle A = 80^0$

(ii)



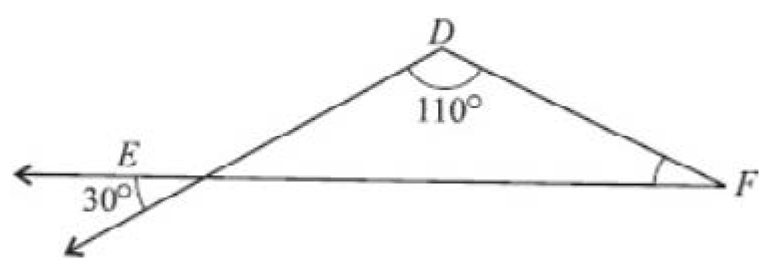

We have,
$m\angle O = 90^0$, $m\angle A = 40^0$, $m\angle B = ?$

We know that

$m\angle O + m\angle A + m\angle B = 180^0$
$90^0 + 40^0 + m\angle B = 180^0$
$130^0 + m\angle B = 180^0$
$m\angle B = 180^0 - 130^0 = 50^0$

Thus, $m\angle B = 50^0$

(iii)



We have, $m\angle D = 110^0$

$m\angle E = 30^0$ (vertically opposite angles are equal)

$m\angle F = ?$

We know that

$m\angle D + m\angle E + m\angle F = 180^0$ (vertically opposite angles are equal)

$110^0 + 30^0 + m\angle F = 180^0$

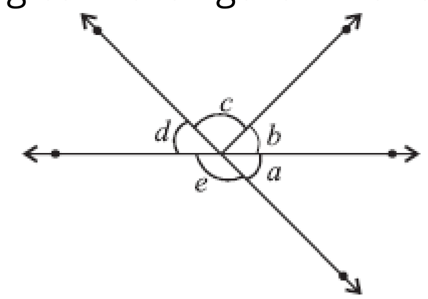
$140^0 + m\angle F = 180^0$

$m\angle F = 180^0 - 140^0 = 40^0$

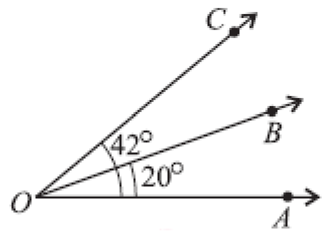
Thus, $m\angle F = 40^0$

## EXERCISE 10.1

1. Name all the angles in the figure which are adjacent.




2. In the following figure $\angle AOB$ and $\angle BOC$ are adjacent angles. i.e. $m\angle AOB = 20^0$ and $m\angle AOC = 42^0$. Find $m\angle BOC$.




3. Identify the pairs of complementary and supplementary angles.

(i) $50^0, 40^0$ (ii) $120^0, 60^0$ (iii) $70^0, 70^0$

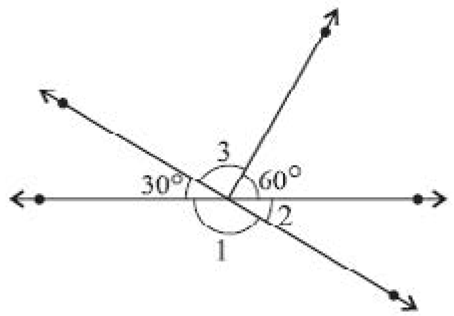
(iv) $130^0, 50^0$ (v) $70^0, 20^0$ (vi) $50^0, 100^0$

4. In the given figure, find all the remaining angles.

$m\angle 1 =$ ______

$m\angle 2 =$ ______

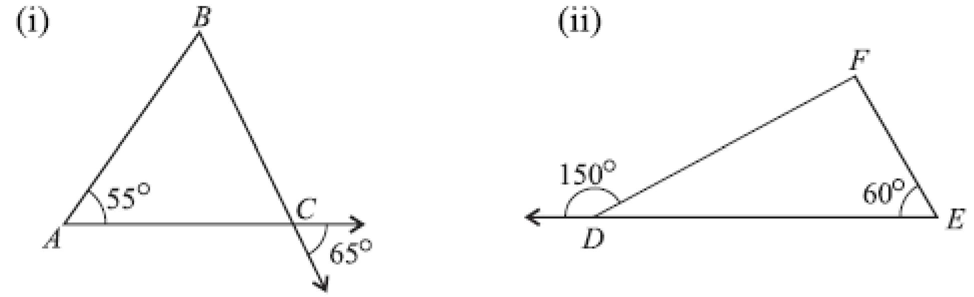
$m\angle 3 =$ ______



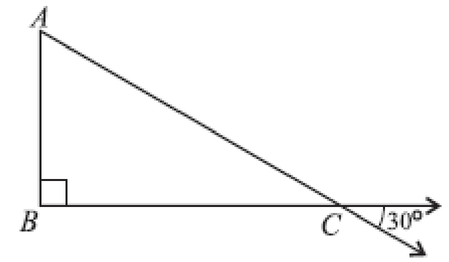





5. Find the unknown angles of the given triangles.




6. Find the remaining angles in the given right angled triangle.

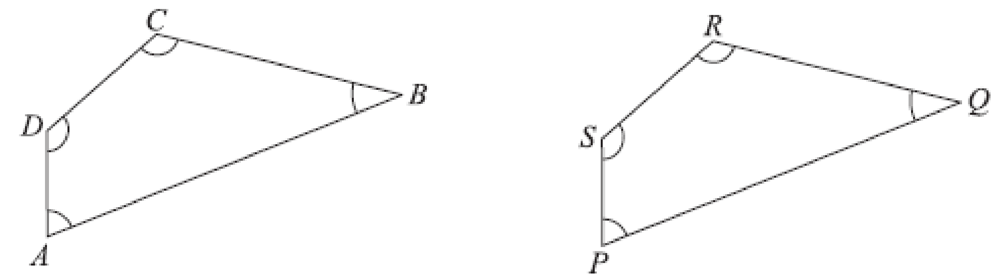



# 10.2 Congruent and Similar Figures

## 10.2.1 Identification of Congruent and Similar Figures

- **Congruent Figures**

Two geometrical figures are said to be congruent, if they have same shape and size. Look at the following figures.




The sides and angles of two given figures can be matched as given below.

$m\overline{AD} = m\overline{PS}$ $m\angle A = m\angle P$

$m\overline{AB} = m\overline{PQ}$ $m\angle B = m\angle Q$

$m\overline{BC} = m\overline{QR}$ $m\angle C = m\angle R$

$m\overline{CD} = m\overline{RS}$ $m\angle D = m\angle S$

From the above, it can be seen that the two figures have exactly the same shape and size. Therefore, we can say that these two figures are congruent.

### 10.2.2 Recognizing the Symbol of Congruency

We have learnt that two geometrical figures are congruent if they have the same shape and same size. The congruency of two figures is denoted by a symbol $\cong$ which is read as "is congruent to". The symbol $\cong$ is made up of two parts. i.e.

- $\sim$ means the same shape (similar). • = means the same size (equal).

The symbol for congruence was developed by Gottfried Leibniz. He was born in 1646 and died in 1716. Gottfried Leibniz made very important contributions to the notation of Mathematics.

- **Similar Figures**

The figures with the same shape but not necessarily the same size are called similar figures. The similarity of the geometrical figures is represented by the symbol "$\sim$". For example, all circles are similar to each other and all squares are also similar.

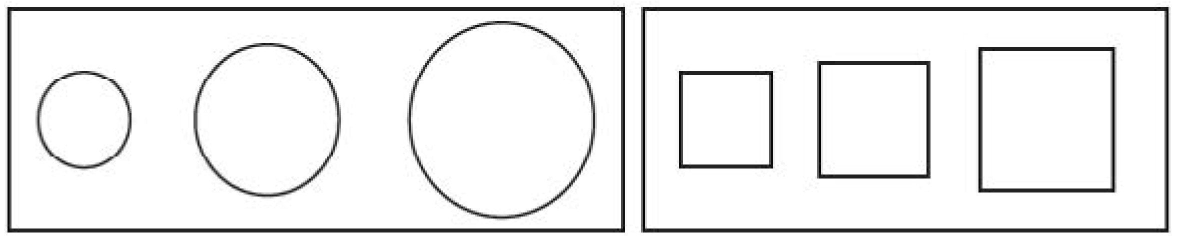


*Figure 1*

But these are not congruent to each other as the size of each circle is different.

### 10.2.3 Applying the Properties for two Figures to the Congruent or Similar

The difference between similar and congruent figures is that:

- Congruent figures have the same shape and same size.
- Similar figures have the same shape, but may be different in sizes.

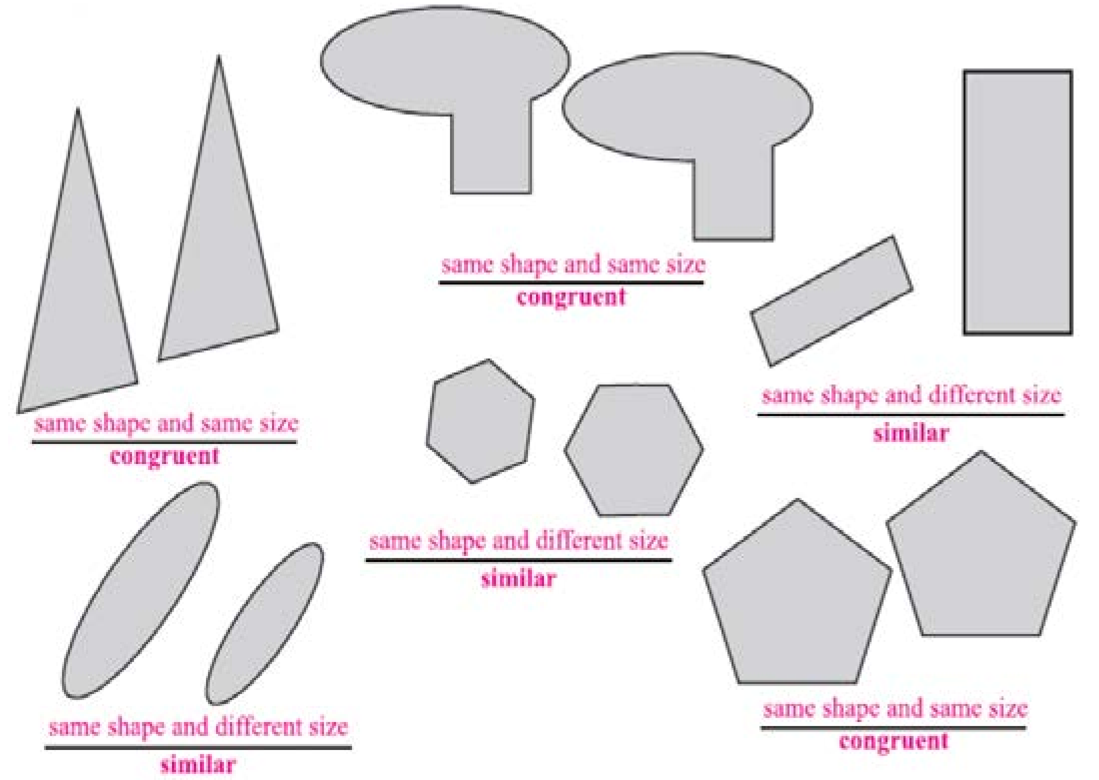
Let us use the same properties for two figures to find whether they are congruent or similar.





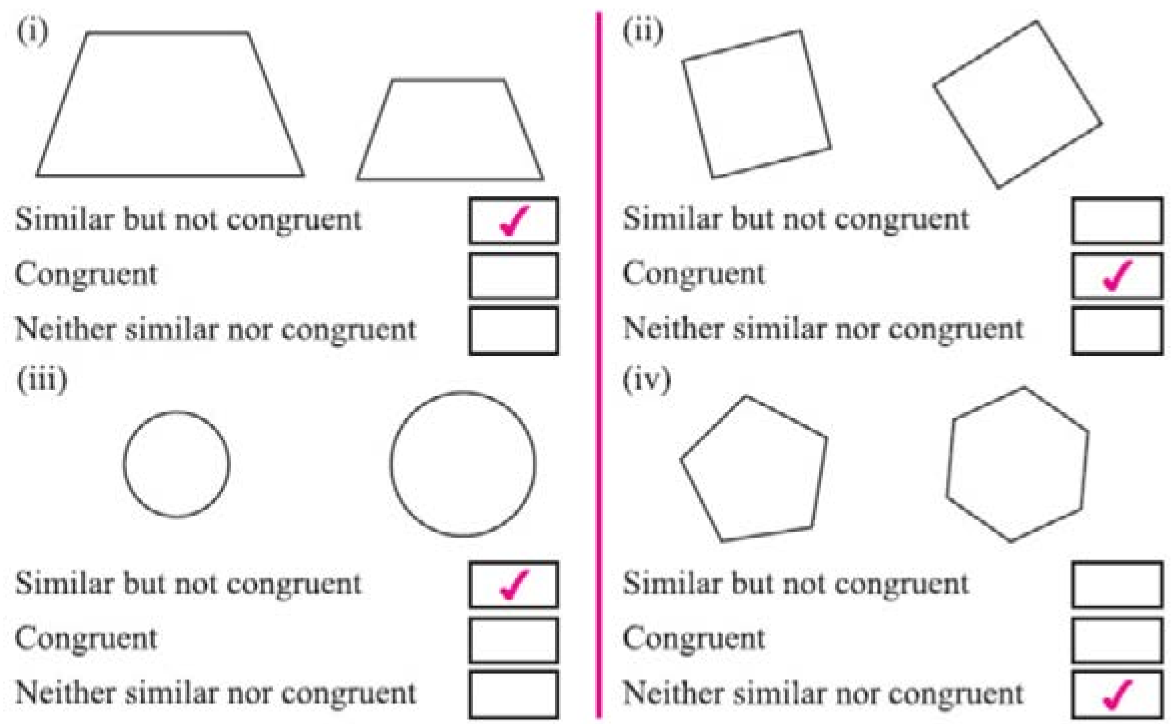
**Example 1:** Decide whether the figures in each pair are congruent or only similar.

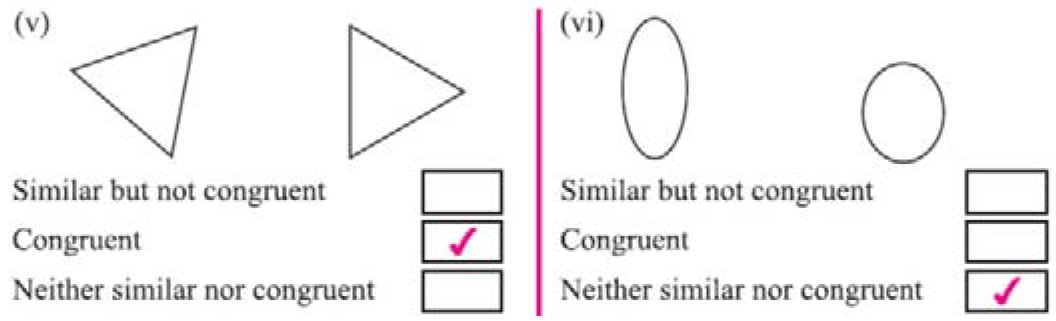
**Solution:**




**Example 2:** How are the following shapes related?

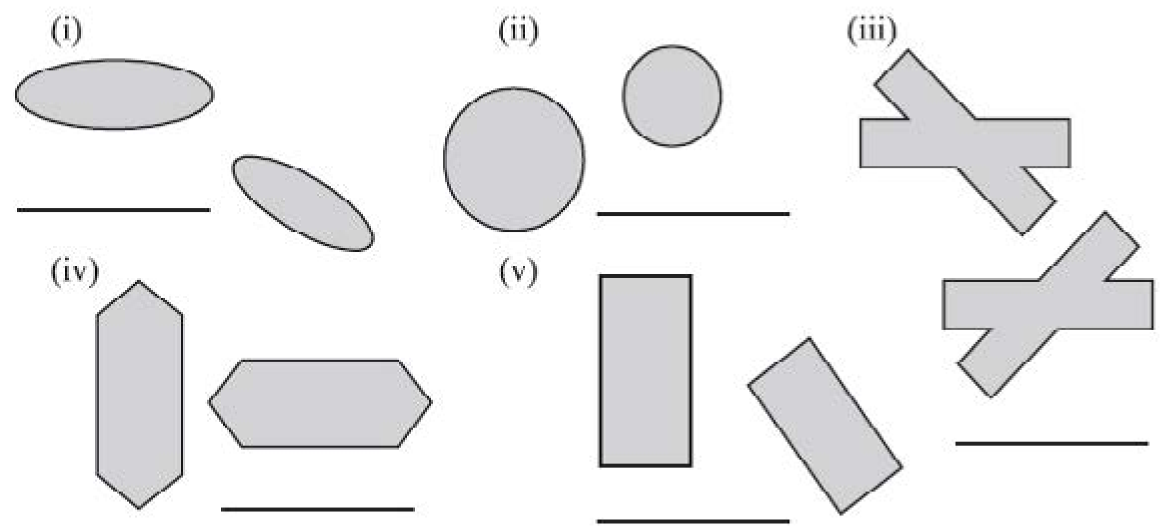
**Solution:**






## EXERCISE 10.2

1. Define similar geometrical figures with examples.
2. Are similar figures congruent? Give examples.
3. Are congruent figures similar? Prove this with examples.
4. Identify congruent and similar pairs of figures.




## 10.3 Congruent Triangles

While considering triangles, two triangles will be congruent if:

a) All the three sides of one triangle are congruent to all three corresponding sides of the other triangle, i.e. SSS $\cong$SSS.





For example, if

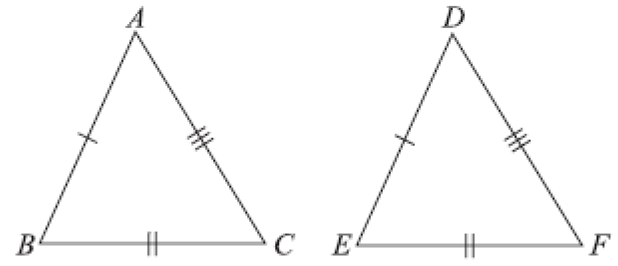
$\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\overline{AC} \cong \overline{DF}$

Then, $\Delta ABC \cong \Delta DEF$




b) Two sides of one triangle and their included angle are congruent to the two corresponding sides and angle of the other triangle, i.e. SAS $\cong$SAS. For example,

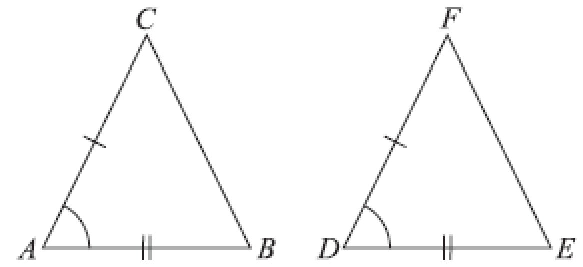
In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$

$\angle A \cong \angle D$

$\overline{AC} \cong \overline{DF}$

Then, $\Delta ABC \cong \Delta DEF$.




c) Two angles of one triangle and their included side are congruent to the two corresponding angles and side of the other triangle, i.e. ASA $\cong$ ASA. For example,

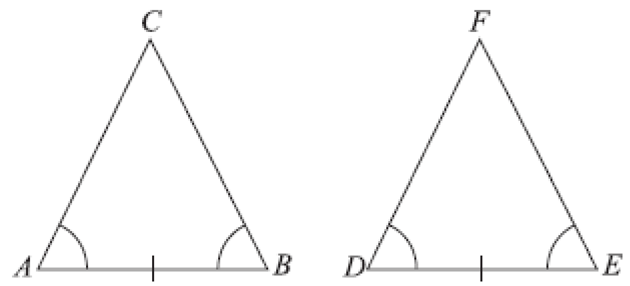
In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle A \cong \angle D$

$\overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$

Then, $\Delta ABC \cong \Delta DEF$



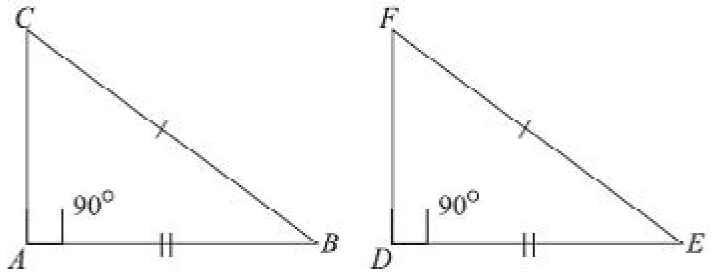

d) The hypotenuse and one side (base or attitude) of a triangle are congruent to the corresponding hypotenuse and one side of the other triangle i.e. RHS $\cong$ RHS. For example, In $\Delta ABC \leftrightarrow \Delta DEF$

$m\angle A = m\angle D = 90^\circ$

$\overline{BC} \cong \overline{EF}$

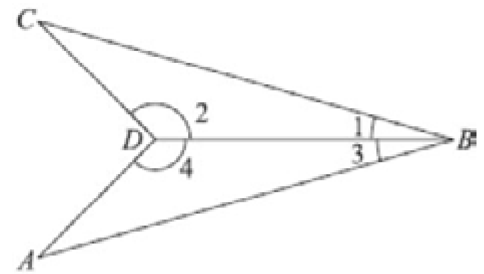
$\overline{AB} \cong \overline{DE}$

Then, $\Delta ABC \cong \Delta DEF$

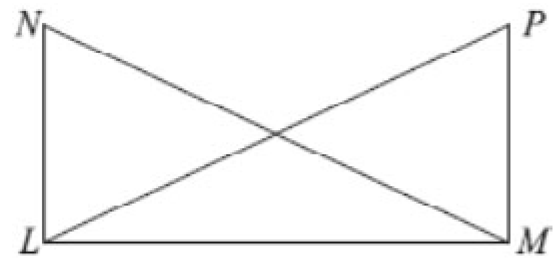



## EXERCISE 10.3

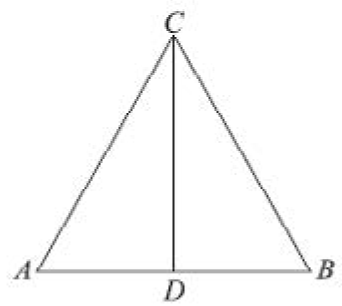
1. If the measures of two angles of a triangle are 35° and 80°, then find the measure of its third angle.
2. In the given figure, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.
   Then prove that, $\triangle ABD \cong \triangle BDC$




3. If the bisector of an angle of a triangle bisects its opposite side, then prove that the triangle is an isosceles triangle.
4. In the given figure, $\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$, then prove that $\angle P \cong \angle N$ and $\angle LMN \cong \angle MLP$



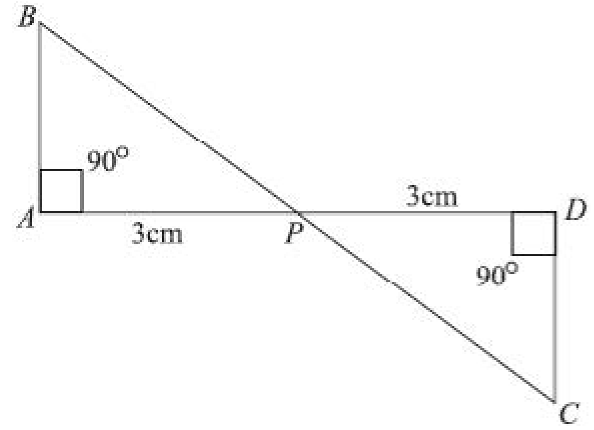

5. In the given triangle $\triangle ABC$, $\overline{CD} \perp \overline{AB}$ and $\overline{CA} \perp \overline{CB}$, then prove that $\overline{AD} \cong \overline{BD}$ and $\angle ACD \cong \angle BCD$







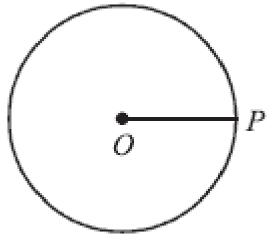

6. Look at the figure to show that $\triangle ABP \cong \triangle DCP$




## 10.4 Circle

A circle is the most familiar shape of the geometry that we often observe around us, a wheel, the Sun, full Moon, coins of one, two and five rupees are some examples of a circle. So, we can define it as:

"A circle is a set of points in a plane which are equidistant from a fixed point, called center of the circle". Let "*P*" be any point which is moving so that it remains at equal distance from a fixed point "*O*". This point will trace a circle whose center will be "*O*" as shown in the figure.

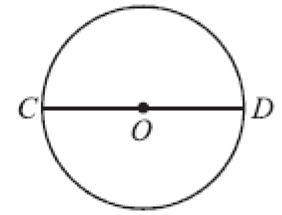



- **Radius**

The distance between the center and any point on the circle is called radius. Here the distance between "*O*" and "*P*" is called radius. In this figure $\overline{OP}$ is the radial segment.

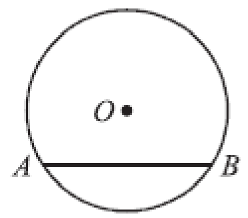
- **Diameter**

A line segment that passes through the center of a circle and touches two points on its edge is called the diameter of the circle. In the given figure, $\overline{AB}$ is the diameter of the circle.



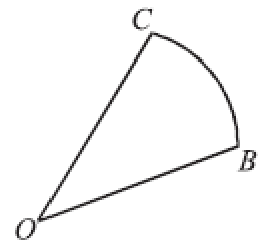
- **Chord**

A line segment joining two points on a circle is called the chord. The figure shows that $\overline{AB}$ is the chord of the circle.




- **Arc**

If we cut a circle it will give us the curved shape as shown in the figure. This whole figure is called sector whereas $\overset{\frown}{BC}$ is called arc of the circle with radii $\overline{OB}$ and $\overline{OC}$




An arc consists of two end points and all the points on the circle between these endpoints. When we cut a circle in such a way that a sector of the circle is smaller than the other, we get two types of arcs, i.e. minor arc and major arc.

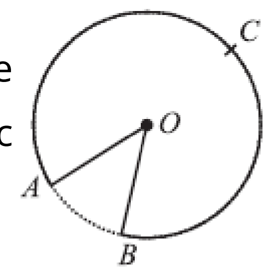
- **Minor Arc**

An arc which is smaller than half of the circle is called minor arc. It is named by using two end points of the arc.

- **Major Arc**

An arc which is more than half of the circle is called major arc. It is named by three points. The first and third arc end points and the middle point is any point on the arc between the end points. For example,

If we cut a circle at any points A and B. It will provide us two arcs $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$. The arc $\overset{\frown}{AB}$ is minor arc whereas a way through $\overset{\frown}{ACB}$ makes a major arc.

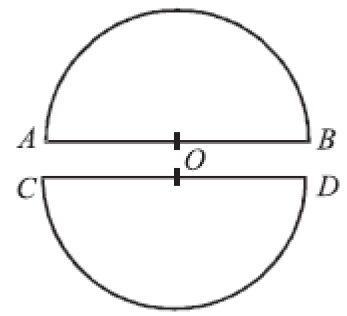



### 10.4.2 Semicircle

Now consider the case, if we cut a circle such that both the arcs are equal. Then it can happen only when it is cut along its diameter. This process generates two semicircles or two half circles.








On joining these semicircles, we get the same circle again. Now let us demonstrate the property of a semicircle that the angle in a semicircle is a right angle.

**Step 1:** Draw a circle, mark its center and draw a diameter through the center. Use the diameter to form one side of a triangle. The other two sides should meet at a vertex somewhere on the circumference.

**Step 2:** Divide the triangle in two by drawing a radius from the center to the vertex on the circumference.

**Step 3:** Recognize that each small triangle has two sides that are radii. All radii are the same in a particular circle. This means that each small triangle has two sides the same length. They must therefore both be isosceles triangles.

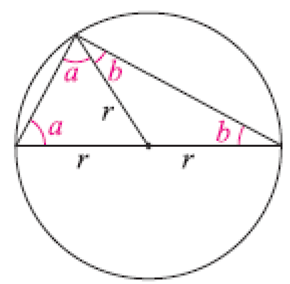
**Step 4:** Because each small triangle is an isosceles triangle, they must each have two equal angles.

**Step 5:** The sum of internal angles in any triangle is 180°. By comparison with the diagram in step 5, we notice that the three angles in the big triangle are *a*, *b* and *a* + *b*. So, we can write an equation as:

$a + b + (a + b) = 180^0$

$2a + 2b = 180^0$

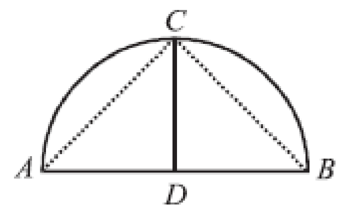
$$a + b = \frac{180^0}{2} = 90^0$$

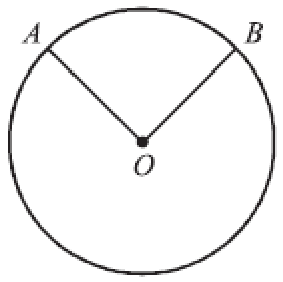



Hence proved, the angle in a semicircle is a right angle.

### 10.4.3 Segment of a Circle

Segment of a circle is a part of circle, cut along any chord. Look at the following figures.

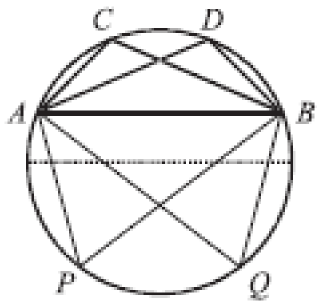



The arc $\overset{\frown}{ACB}$ corresponding to the chord $\overset{\frown}{AB}$ is called segment of the circle. In second figure *AOB* is the sector of the circle.

Now cut the circle along a chord other then diameter. You have two segments. Now draw two inscribed angles of the smaller segment $\angle ACB$ and $\angle ADB$. Now measure them, we will see that both angles are equal in measurement.

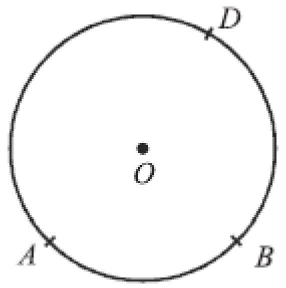
i.e. $m\angle ACB = m\angle ADB$.




Again draw the two angles in the major segments of the circle, $\angle APB$ and $\angle AQB$. Measure them again, have you noticed that again $m\angle APB = m\angle AQB$. Therefore, we can draw a conclusion; the angles in the same segment of a circle are equal.

## EXERCISE 10.4

1. Draw a circle with radius $\overline{OA} = 5cm$ and find its diameter.
2. In the given figure locate major and minor arcs.




3. How many diameters of a circle can be drawn. Draw a circle and trace at least '5' different diameters.





4. Draw a semicircle of a radius of 8cm.
5. Draw a circle and cut it into two segments. Construct two inscribed angles in each of the segment and measure them.

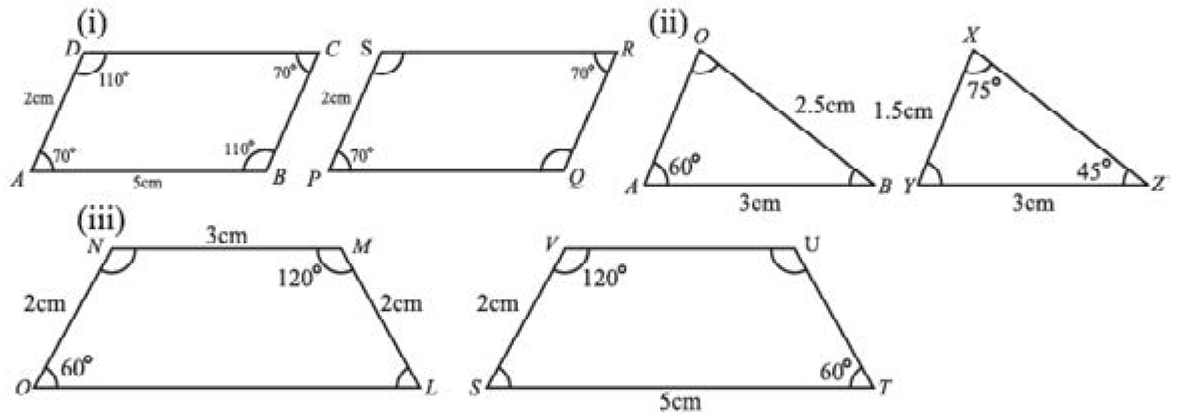
## REVIEW EXERCISE 10

1. Answer the following questions.
   (i) What is meant by the adjacent angles?
   (ii) What is the difference between complementary angles and supplementary angles?
   (iii) Define the vertically opposite angles.
   (iv) What is the symbol of congruency?
   (v) What is a circle?
   (vi) Differentiate between major and minor arcs.

2. Fill in the blanks.
   (i) From the ______ angles we mean, angles next to each other.
   (ii) If the sum of two angles is ______ then the angles are called complementary angles.
   (iii) The non-adjacent angles which are formed from two intersecting lines are called ______ angles.
   (iv) Two figures are congruent if they are same in______ and ________.
   (v) A circle is a set of points which are equidistant from a fixed point, called its ______.
   (vi) Two triangles are congruent, if three sides of one triangle are ______ to the three sides of other triangle.
   (vii) The figures with same shape but not necessarily the same size are called ______ figures.
   (viii) Vertically opposite angles are always ______ in measure.

3. Tick ( ✓ ) the correct option.

4. Find unknown measures of the sides and angles for these congruent shapes.




5. If a and b are complementary angles, then find the value of '*b*' if measure of '*a*' is 40°.
6. If $x$ and $y$ are two supplementary angles where as $m\angle x = 60^0$. Then find the measure of $y$.

## SUMMARY

- Two angles with a common vertex, one common arm and uncommon arms on opposite sides of the common arm, are called adjacent angles.
- If the sum of two angles is 90°, then the angles are called complementary angles.
- If the sum of two angles is 180°, then the angles will be supplementary to each other.
- If two lines intersect each other, the non adjacent angles, so formed are called vertical angles.
- Two geometrical figures are similar if they are same in shape.
- Figures are congruent if they are same in shape and size.
- Two triangles will be congruent if any of the following property holds.
  (i) $SSS \cong SSS$ (ii) $SAS \cong SAS$ (iii) $ASA \cong ASA$ (iv) $RHS \cong RHS$
- A path traced by a point remaining equidistant from the fixed point, generates circle.
- A line segment joining two points on a circle is called chord of a circle.





- A segment of a circle cut across diameter is called semicircle.
- An angle in a semicircle is a right angle.
- The angles in the same segment of a circle are equal.

CHAPTER

# 11 PRACTICAL GEOMETRY

*Animation 11.1: Parallelogram Area*
*Source & Credit: .wikipedia*

## Student Learning Outcomes

**After studying this unit, students will be able to:**

- Divide a line segment into a given number of equal segments.
- Divide a line segment internally in a given ratio.
- Construct a triangle when perimeter and ratio among the lengths of sides are given.
- Construct an equilateral triangle when
  - base is given
  - altitude is given
- Construct an isosceles triangle when
  - base and a base angle are given,
  - vertex angle and altitude are given,
  - altitude and a base angle are given.
- Construct a parallelogram when
  - two adjacent sides and their included angle are given,
  - two adjacent sides and a diagonal are given.
- Verify practically that the sum of.
  - measures of angles of a triangle is 180°
  - measures of angle of a quadrilateral is 360°

## 11.1 Line Segment

We know that we can compare two line segments by measuring their lengths.

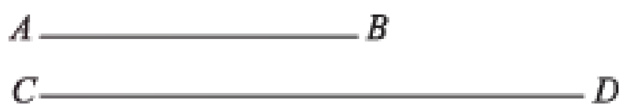


Figure (i)

In figure (i), we can see that the line segment $\overline{AB}$ is shorter than $\overline{CD}$ because the length of $\overline{AB}$ is less than that of $\overline{CD}$ i.e. $m\overline{AB} < m\overline{CD}$

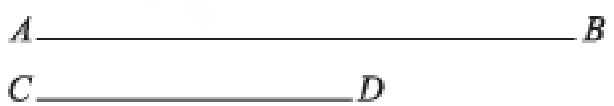


Figure (ii)





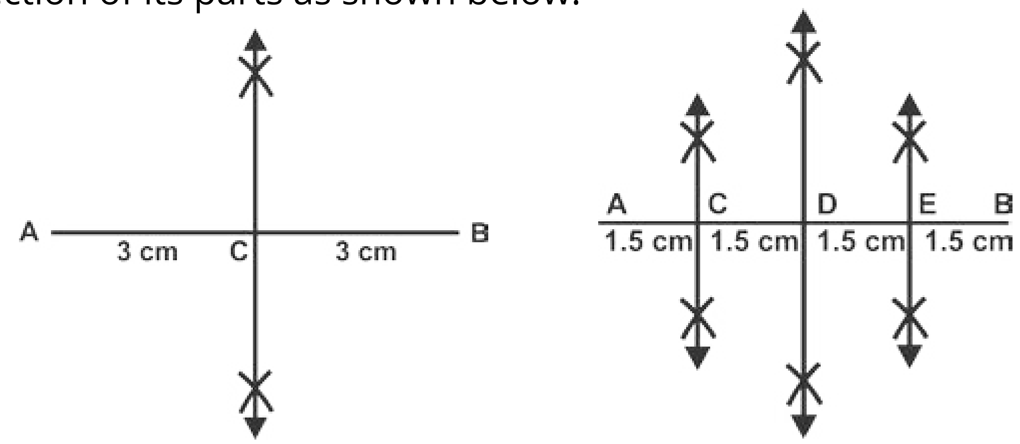
In figure (ii), we can notice that the line segment $\overline{AB}$ is longer than $\overline{CD}$ because the length of $\overline{AB}$ is greater than that of $\overline{CD}$ i.e. $m\overline{AB} > m\overline{CD}$.

Figure (iii)

In figure (iii), we can check the third and final possibility of the comparison of two line segments. Here we can see that two line segments are equal in length, i.e. $m\overline{AB} = m\overline{CD}$
Such line segments which have equal lengths are called congruent line segments.

### 11.1.1 Division of a Line Segment into Number of Equal Segments

In our previous class, we have learnt that a fine segment can be divided into an even number of line segments by successive bisection of its parts as shown below:




Now we learn a method for dividing a line segment into an odd number of congruent parts, i.e. 3, 5, 7, ........ and so on. We shall learn this method with the help of an example.

**Example 1:** Divide a line segment of length 14cm in 7 equal line segments.

**Solution:**

**Steps of construction:**

(i) Draw a 14cm long line segment *PQ*. (Use a ruler)

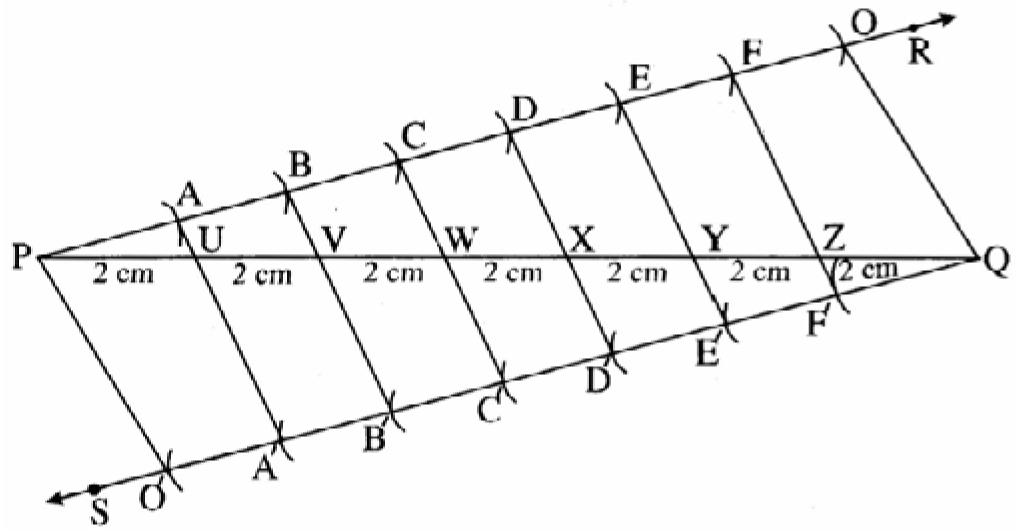
(ii) Draw a ray *PR* making an acute angle with a line segment *PQ*. (Use a ruler)

(iii) Draw another ray *QS* making the same acute angle with $\overline{PQ}$

(iv) Draw 7 arcs (according to the required parts of a line segment) of suitable radius, intersecting the ray *PR* at points *A, B, C, D, E, F* and *O* respectively. (Start from *A* and for each arc consider the previous point as the starting point).

(v) Similarly, draw 7 arcs of same radius, intersecting the ray *QS* at points *F′, E′, D′, C′, B′, A′* and *O′* respectively.

(vi) Draw line segments *PO′, AA′, CC′, DD′, EE′, FF′* and *OQ*. These line segments intersect the line segment *PQ* at points *U, V, W, X, Y* and *Z* respectively.




(vii) Hence $\overline{PU}, \overline{UV}, \overline{VW}, \overline{WX}, \overline{XY}, \overline{YZ}$ and $\overline{ZQ}$ are the required 7 congruent parts of line segment $\overline{PQ}$

| **Note:** | Result can be checked by measuring the each part with a divider. |
|---|---|

## 11.1.2 Division of a Line Segment in a given Ratio

In previous example, we can notice that the intersecting points *U, V, W, X, Y* and *Z* are also dividing the $\overline{PQ}$ in a specific ratio.

- The point *U* is dividing the line segment *PQ* in ratio 1 : 6.
- The point *V* is dividing the line segment *PQ* in ratio 2 : 5.





- The point *W* is dividing the line segment *PQ* in ratio 3 : 4.
- The point *X* is dividing the line segment *PQ* in ratio 4 : 3.
- The point *Y* is dividing the line segment *PQ* in ratio 5 : 2.
- The point *Z* is dividing the line segment *PQ* in ratio 6 : 1.

Now we learn the division of a line segment in a given ratio. Suppose that the given ratio is *a: b: c*. So,

**Step 1:** Draw a line segment *PQ*.

**Step 2:** Draw two rays *PR* and *QS* making acute angles with line *PQ*.

**Step 3:** Draw *a* + *b* + *c* number of arcs at equal distance on $\overline{PR}$ and $\overline{QS}$.

**Step 4:** Join the points of $\overline{PR}$ and $\overline{QS}$ corresponding to the ratio *a : b : c*.

The intersecting points divide the line segment *PQ* in a given ratio *a : b : c*.

**Example 2:** Divide a long line segment of length 12cm in the ratio 1 : 3 : 4.

**Solution:**

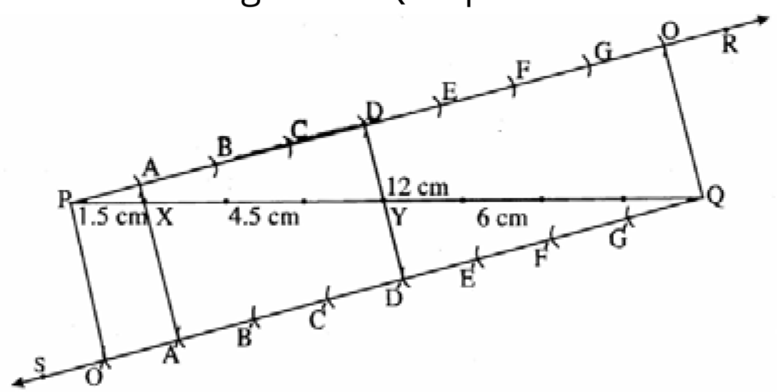
**Steps of construction:**

(i) Draw a 12cm line segment *PQ*. (Use a ruler)

(ii) Draw two rays *PR* and *QS* making same acute angle with line segment *PQ*.

(iii) Draw 1 + 3 + 4 = 8 arcs of suitable radius, intersect the ray *PR* at points *A, B, C, D, E, F, G* and *O* and intersect the ray *QS* at point *G′, F′, E′, D′, C′, B′, A′* and *O′*.

(iv) Draw line segments *PO′*, *AA′*, *DD′* and *OQ′*, these line segments intersect the line segment *PQ* at points *X* and *Y*.



(v) The line segments *PX, XY* and *YQ* are three parts of a line segment *PQ* which are dividing it in the ratio 1 : 3 : 4.

### EXERCISE 11.1

1. Divide a line segment of length 6cm into 3 congruent parts.
2. Divide a line segment of length 7.5cm into 5 congruent parts.
3. Draw a line segment of length 10.8cm and divide it into 6 congruent parts.
4. Divide a line segment of length 10cm into 5 congruent parts.
5. Draw a line segment of length 9.8cm and divide it into 7 congruent parts.
6. Divide the line segment:
   a. $\overline{AB}$ of length 4cm in the ratio 1 : 2.
   b. $\overline{PQ}$ of length 7.5cm in the ratio 2 : 3.
   c. $\overline{XY}$ of length 9cm in the ratio 2 : 4.
   d. $\overline{DE}$ of length 6cm in the ratio 1 : 2 : 3.
   e. $\overline{DE}$ of length 6cm in the ratio 1 : 1 : 2.
   f. $\overline{LM}$ of length 13.5cm in the ratio 2 : 3 : 4.
   g. $\overline{UV}$ of length 11.2cm in the ratio 1 : 2 : 4.

## 11.2 Triangles

We are already familiar with the different methods of triangle construction. Here we shall learn more methods.

### 11.2.1 Construction of a Triangle when its Perimeter and Ratio among the Lengths of Sides are Given

A triangle can also be constructed if we have the perimeter of a triangle and the ratio among the lengths of its sides.





**Example 1:** Construct a triangle whose perimeter is 12cm and 2:3:4 is the ratio among the lengths of its sides.

**Solution:**

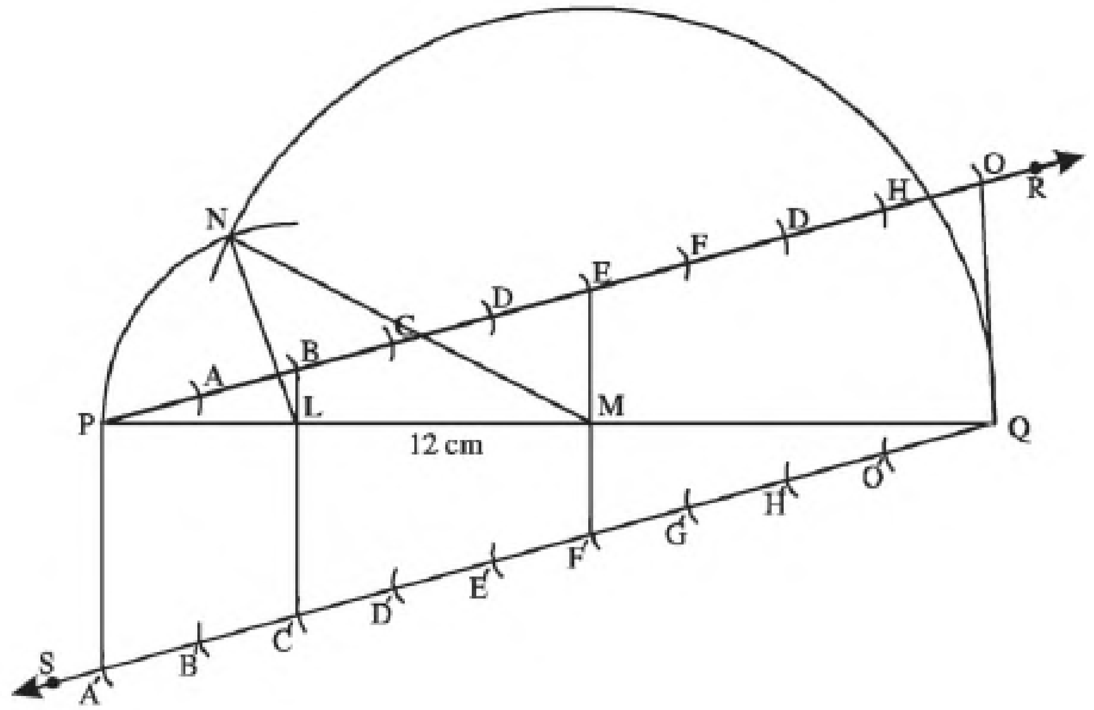
**Steps of construction:**

(i) Draw a line segment *PQ* of length 12cm. (use a ruler)

(ii) Divide the line segment *PQ* in the given ratio 2:3:4.

(iii) Consider the point *L* as center and draw an arc by using the length of *PL* as radius.

(iv) Again consider the point *M* as center and draw another arc by using the length of *MQ* as radius.




(v) Label the point of intersection of two arcs as *N*.

(vi) Join the point *N* to *L* and *M* respectively.

$\Delta LMN$ is the required triangle.

### 11.2.2 Construction of Equilateral Triangles

An equilateral triangle is a triangle in which all three sides are equal and all three angles are congruent. It can be constructed using a given length of a line segment (base and altitude). Let us construct an equilateral triangle when:

- **Base is Given**

Here it begins with the given base which is the length of each side of the required equilateral triangle. Let us make it clear with an example.

**Example 2:** Construct an equilateral triangle ∆ABC whose base is 4.5cm long.

**Solution:**

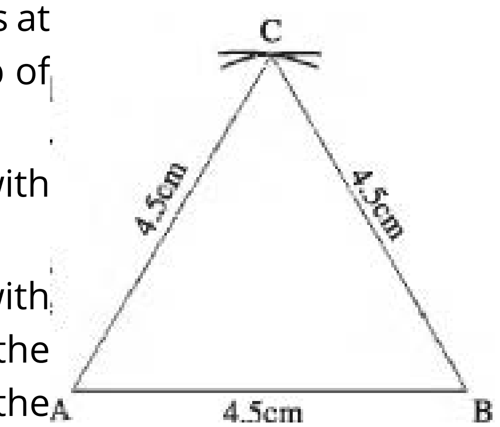
**Steps of construction:**

(i) Draw a line segment of length 4.5cm using a ruler. Label its end points *A* and *B*.

(ii) Place the needle of the compasses at point *A* and open it so that the tip of the pencil touches the point *B*.

(iii) Draw an arc of radius $\overline{AB}$ with centre *A*.

(iv) Draw another arc of radius $\overline{AB}$ with centre *B*. This arc will intersect the first arc at one point. Name the meeting point of two arcs as *C*.




(v) Finally join the point *C* with the point *A* and with the point *B*. The triangle *∆ABC* is the required equilateral triangle.

- **Altitude is Given**

An equilateral triangle can also be constructed if its altitude is given. Let us learn this method with an example.

**Example 3:** Construct an equilateral triangle *∆XYZ* whose altitude is of measure 5cm.

**Solution:**

**Steps of construction:**

(i) Draw a line AB using a ruler and mark any point P on it.

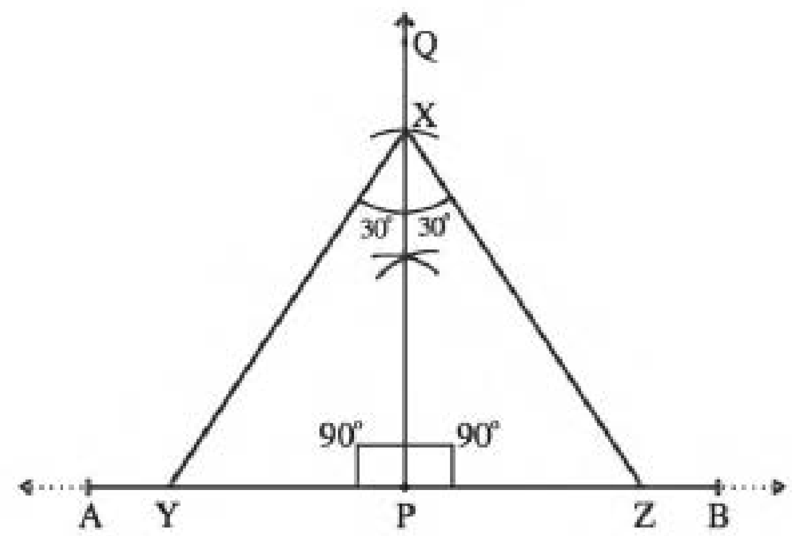
(ii) Draw a perpendicular $\overrightarrow{PQ}$ on line *AB*, i.e. $\overrightarrow{PQ} \perp \overleftrightarrow{AB}$.

(iii) From point *P* draw an arc of measure 5cm. This arc will cut the perpendicular *PQ* at the point *X* as shown.





(iv) Construct the angles of 30° at point *X* i.e. m∠*PXY* = 30° and m∠*PXZ* = 30°.




*∆XYZ* is the required equilateral triangle.

## 11.2.3 Construction of Isosceles Triangles

As isosceles triangle is a triangle in which two sides are equal in length. These two sides are called legs and third side is called the base. The angles related to the base are also congruent. An isosceles triangle can be constructed when:

- **Base and a Base Angle are Given**

We know that the base angles of an isosceles triangle are always equal. So, we can construct an isosceles triangle with the measure of its base and base angle are given.

**Example 4:** Construct an isosceles triangle ∆ *LMN* whose base is of measure 6cm and measure of base angle is 30°.

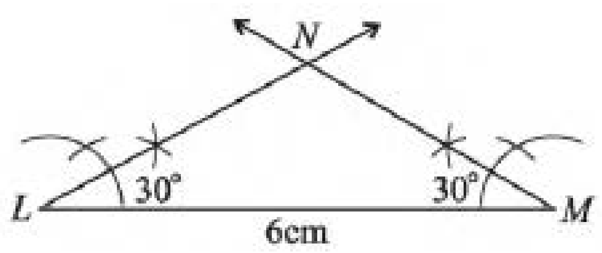
**Solution:**

**Steps of construction:**

(i) Draw a line segment $\overline{LM}$ of length 6cm. (By using a ruler)

(ii) Construct an angle m∠*MLN* = 30° at the point *L*.

(iii) Construct another angle *LMN* of 30° at point *M*. The produced arms of these angles intersect at point *N*.




$\Delta LMN$ is the required isosceles triangle.

- **Vertex Angle and Altitude are Given**

In an isosceles triangle, the angle formed by the two sides of equal length and opposite to the base is called vertex angle. When the altitude is drawn to the base of an isosceles triangle, it bisects the vertex angle. This property can be used to construct an isosceles triangle with its vertex angle and measure of altitude are given.

**Example 5:** Construct an isosceles triangle with altitude = 3.5cm and vertex angle = 50°

**Solution:**

**Steps of construction:**

(i) Draw a line *XY* and choose any point *D* on it.

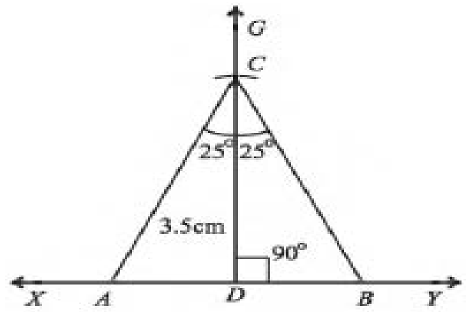
(ii) Draw a perpendicular $\overrightarrow{DG}$ above the base line.

(iii) From point *D*, draw an arc of radius 3.5cm to cut this perpendicular at point C.

(iv) Since the vertex angle is 50° and an altitude bisects it. So, perpendicular will show the angle of 25° on both sides,

i.e. $\frac{50^0}{2} = 25^0$

(v) Draw two arms making an angle 25° with perpendicular *CD,* on both sides and let these arms cut the base line at points *A* and *B.*




$\Delta ABC$ is the required isosceles triangle.





- **Altitude and Base Angle are Given**

We know that in an isosceles triangle, base angles are congruent and sum of three angles is 180°. It means, if the base angle of an isosceles triangle is given, we can find its vertex angle as shown below.

Let the base angle be 40° and vertex angle be *x*. Then according to the isosceles triangle:

$40° + 40° + x = 180°$

$80° + x = 180°$

$x = 180° – 80°$

$x = 100°$

Thus, the vertex angle is of measure 100°.

**Example 6:** Construct an isosceles triangle with altitude = 4cm and base angle = 50°.

**Solution:**

**Steps of construction:**

(i) We know that:

50° + 50° + vertex angle = 180°

100° + vertex angle = 180°

vertex angle = 180° – 100° = 80°

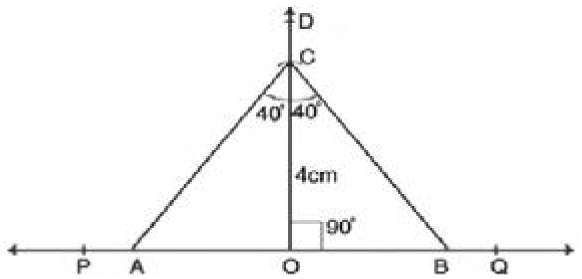
(ii) Draw a line *PQ* and choose any point *O* on it.

(iii) Draw the perpendicular *OD* above the base line.

(iv) From point *O*, draw an arc of radius 4cm and cut the perpendicular at point *C*.

(v) Draw two arms making an angle of $\frac{50^0}{2} = 25^0$ at point *C* on both sides of the perpendicular line $\overline{OD}$.

(vi) Let the arms touch the base line at points *A* and *B*.




$\Delta$ *ABC* is required isosceles triangle.

## EXERCISE 11.2

1. Construct the equilateral triangles of given measures.
   (i) base = 4cm (ii) altitude = 6cm
   (iii) altitude = 5.5cm (iv) base = 3.5cm

2. Construct an isosceles triangle whose:
   (i) base = 3cm and base angle = 45°
   (ii) altitude = 4.8cm and vertex angle = 100°
   (iii) base = 5cm and base angle = 65°
   (iv) altitude = 4.2cm and base angle = 35°

3. Construct a triangle $\Delta LMN$ whose ratio among the lengths of its sides is 2 : 3 : 4 and perimeter is 10cm.

4. Construct a triangle $\Delta XYZ$ whose perimeter is 13cm and 3 : 4 : 5 is the ratio among the length of its sides.

5. The perimeter of a $\Delta XYZ$ is 12cm and ratio among the lengths of its sides is 4 : 2 : 3. Construct the triangle $\Delta XYZ$.

## 11.3 Parallelogram

A parallelogram is a four-sided closed figure with two parallel and congruent *(equal in measurement)* opposite sides. The opposite angles of a parallelogram are also congruent and its diagonals bisect each other as shown in the figure (a).

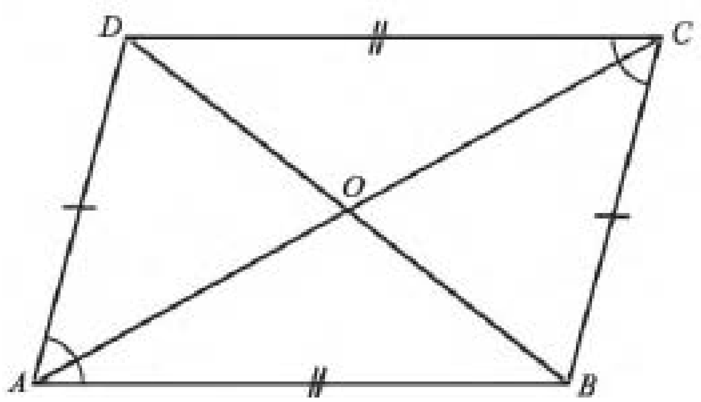



figure (a)





In the above figure (a), we can see that,

(i) $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$
(ii) $m\angle AB = m\angle CD$ and $m\angle AD = m\angle BC$
(iii) $m\angle DAB = m\angle DCB$. i.e. $\angle DAB = \angle DCB$
and $m\angle CDA = m\angle CBA$. i.e. $\angle CDA = \angle CBA$

### 11.3.1 Construction of Parallelogram when two Adjacent Sides and their Included Angle are Given

We can construct a parallelogram when the measurements of two adjacent sides and the related angle which is formed by two sides, are given.

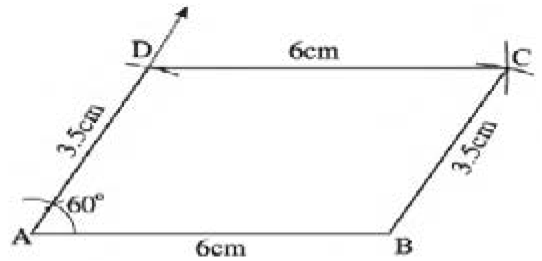
**Example:** Construct a parallelogram *ABCD* if,

$$m\overline{AB} = 6cm \qquad m\overline{AD} = 3.5cm \qquad m\angle A = 60^0$$

**Solution:**

**Steps of construction:**

(i) Draw $\overline{AB}$ 6cm long.
(ii) Construct an angle of 60° at point *A* i.e. $m\angle A = 60^0$.
(iii) Draw an arc of radius 3.5cm.
(iv) Now take the point *B* as center and draw another arc of radius 3.5cm.
(v) Now again consider the point *D* as center and draw an arc of radius 6cm. This arc will intersect the previous arc on the point *C*.
(vi) Join the point *C* and point *D* and also join the point *C* and point *B*.




**Result:** *ABCD* is the required parallelogram.

## EXERCISE 11.3

1. Construct the parallelogram *ABCD* where
 $m\overline{AB} = 7cm \quad m\overline{BC} = 4cm \quad m\angle ABC = 60^0$
2. Construct the parallelogram *PQRS* where
 $m\overline{PQ} = 8cm \quad m\overline{QR} = 4cm \quad m\angle PQR = 75^0$
3. Construct the parallelogram *LMNO* where
 $m\overline{LM} = 6.5cm \quad m\overline{MN} = 4.5cm \quad m\angle LMN = 45^0$
4. Construct the parallelogram *BSTU* where
 $m\overline{BS} = 7.7cm \quad m\overline{ST} = 4.4cm \quad m\angle BST = 30^0$
5. Construct the parallelogram *OABC* where
 $m\overline{OA} = 6.3cm \quad m\overline{AB} = 3.1cm \quad m\angle OAB = 70^0$
6. Construct the parallelogram *DBAS* where
 $m\overline{BA} = 9cm \quad m\overline{AS} = 2.8cm \quad m\angle DBA = 40^0$

- **Construction of Parallelogram when two Adjacent sides and a Diagonal are given:**

A parallelogram can be constructed when we have two adjacent sides and one diagonal as given in the example.

**Example:** Construct the parallelogram *ABCD* if,

$m\overline{AB} = 4cm \quad m\overline{BC} = 3cm \quad m\overline{CA} = 6cm$

**Solution**:

We can examine that $\overline{AB}$ and $\overline{BC}$ are two sides because both have a common point *B* and diagonal *AC* is also bigger in measurement.

**Steps of construction:**

(i) Draw a $\overline{AC}$ of measure 6cm.

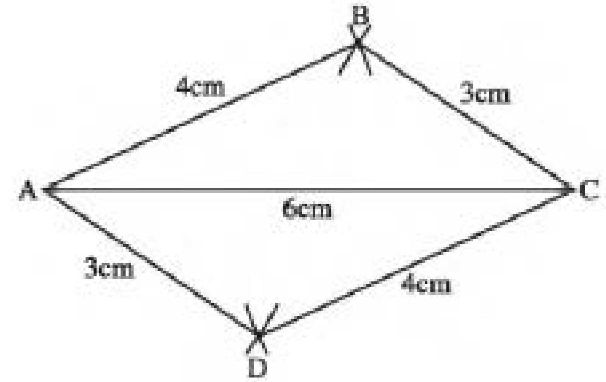
(ii) Consider the point *A* as centre and draw an arc of radius 4cm on the upper side of line segment $\overline{AC}$ and draw another arc radius of 3cm on the lower side of line segment *AC*.





(iii) Now consider the point *C* as centre and draw an arc of radius 3cm on the upper side of segment *AC* and draw another arc of radius 4cm on the lower side of *AC*. (These points of arcs intersect at point *B* and *D*).

(iv) Finally join the points *B* and *D* with the point *A* and then with the point *C*.




Thus, *ABCD* is the required parallelogram.

### 11.3.2 Sum of measure of Angles of a Triangle and a Quadrilateral

- **The sum of Measures of Angles of a Triangle is 180°**

In any triangle, the sum of measures of its angles is 180°. It can be verified as given below.

**Verification:** Let $\Delta ABC$ be a triangle, then according to the given statement, we have to verify that.

$m\angle ACB + m\angle ABC + m\angle BAC = 180°$

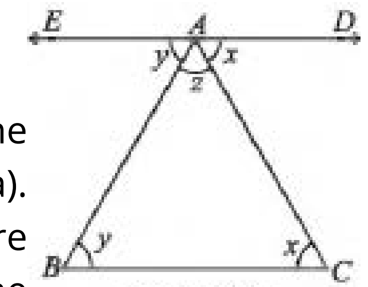


figure (a)

**Step 1:** Draw a line *ED* parallel to the line segment *BC* as shown in the figure (a).

**Step 2:** Since line *ED* and line segment *BC* are parallel. So, according to the properties of parallel lines,

$m\angle ACB = m\angle CAD$

$m\angle ABC = m\angle BAE$

**Step 3:** Label two equal angles ($\angle ACB$ and $\angle CAD$) as *x* and label other two equal angles ($\angle ABC$ and $\angle BAE$) as *y*. Finally, label the angle $\angle BAC$ as *z*.

**Step 4:** It can be seen that the sum of measurement of three angles *x*, *y* and *z* is 180° because these angles are on a straight line i.e.

$$m\angle x + m\angle y + m\angle z = 180°$$

Hence Verified $m\angle ACB + m\angle ABC + m\angle BAC = 180°$

- **The sum of Measures of Angles in a Quadrilateral is 360°**

We have learnt that the sum of three angles in a triangle is 180°. Let us use the same fact to verify that the sum of angles in a quadrilateral is 360°.

**Verification:** Let *ABCD* be a quadrilateral, then we have to verify that, $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

**Step 1:** Join the point *B* with point *D* as shown in the figure (b). This will divide the quadrilateral into two triangles, i.e.

$\Delta ABD$ and $\Delta BCD$.

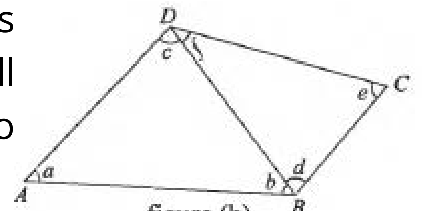


figure (b)

**Step 2:** The sum of angles in a triangle is 180°.

So, In triangle $\Delta ABD$, we have

$$m\angle a + m\angle b + m\angle c = 180°$$

In triangle $\Delta BCD$, we have

$$m\angle d + m\angle e + m\angle f = 180°$$

**Step3:** Find the sum of all the angles of the quadrilateral as,

$$m\angle a + m\angle b + m\angle c + m\angle d + m\angle e + m\angle f = 180° + 180°$$

$$m\angle a + (m\angle b + m\angle d) + m\angle e + (m\angle c + m\angle f) = 360°$$

Hence verified, $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

## EXERCISE 11.4

1. Construct the parallelogram *MNAR* where
$m\overline{MN} = 5cm$  $m\overline{MA} = 2.8cm$  $m\overline{NA} = 7cm$
2. Construct the parallelogram *DGRP* where
$m\overline{DG} = 5.5cm$  $m\overline{GP} = 1.9cm$  $m\overline{DP} = 6.8cm$





3. Construct the parallelogram *ABCD* where
$m\overline{AD} = 3.1cm$  $m\overline{GP} = 6.5cm$  $m\overline{DP} = 8cm$
4. Construct the parallelogram *VTSE* where
$m\overline{SR} = 1.5cm$  $m\overline{RT} = 3.6cm$  $m\overline{TS} = 4.8cm$
5. Construct the parallelogram *DBCO* where
$m\overline{BC} = 4.4cm$  $m\overline{BO} = 6.6cm$  $m\overline{CO} = 7.7cm$
6. Construct the parallelogram *MASK* where
$m\overline{MA} = 3.1cm$  $m\overline{AS} = 6.4cm$  $m\overline{MS} = 5.2cm$

## REVIEW EXERCISE 11

1. Answer the following questions.
   (i) Which line segments are called congruent line segments?
   (ii) Write the sum of interior angles of a triangle.
   (iii) Define an equilateral triangle.
   (iv) Name the equal sides of an isosceles triangle.
   (v) What is meant by the vertex angle in an isosceles triangle?

2. Fill in the blanks.
   (i) An _______ triangle can be constructed if the length of its one side is given.
   (ii) We compare two line segments by measuring their _________.
   (iii) Two line segments of an _______ length are called congruent line segments.
   (iv) A polygon with three sides and three vertices is called a ________ .
   (v) The opposite angles of a parallelogram are also _______.
   (vi) Two equal sides of an isosceles triangles are called _______ and 3rd side is called the _______.

3. Tick ( ✓ ) the correct option.

4. Divide a line segment of length 9.8cm into 7 congruent parts.
5. Divide a line segment $\overline{LM}$ of length 13.5cm in the ratio 2:3:4.
6. Construct an equilateral triangle whose altitude is 3.8cm.
7. Construct an isosceles triangle whose altitude is 5cm and base angles are $67\frac{1}{2}^{0}$
8. Construct a parallelogram *ABCD*, if:
$m\overline{AB} = 5.4cm$ $m\overline{BC} = 2.4cm$ $m\overline{AC} = 6.6cm$

## SUMMARY

- The line segments having an equal length are called congruent line segments.
- A triangle is a polygon with three sides and three vertices and its sum of interior angles is 180°.
- A triangle can also be constructed with its perimeter and ratio among lengths of its sides.
- An equilateral triangle is a triangle in which all three sides are equal and all three angles are congruent.
- An isosceles triangle is a triangle in which two sides are equal and base angles are congruent.
- A parallelogram is a four sided closed figure with two parallel and congruent opposite sides.

CHAPTER

12

# CIRCUMFERENCE, AREA AND VOLUME

## Student Learning Outcomes

**After studying this unit, students will be able to:**

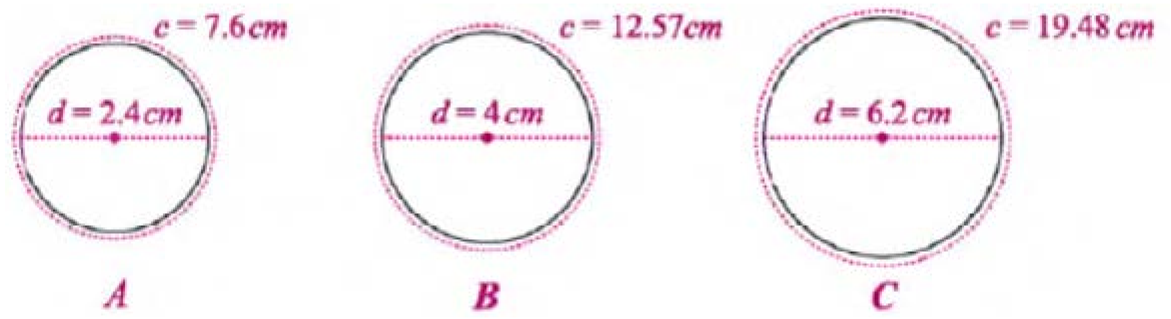
- Express $\pi$ as the ratio between the circumference and the diameter of a circle.
- Find the circumference of a circle using formula.
- Find the area of a circular region using formula.
- Find the surface area of a cylinder using formula.
- Find the volume of cylindrical region using formula.
- Solve real life problems involving;
  - circumference and area of a circular region.
  - surface area and volume of a cylinder.

# 12.1 Circumference, Area and Volume

## 12.1.1 Expressing $\pi$ as the Ratio between Circumference of a Circle and diameter

The circumference of a circle is the distance around the edge of the circle. It could be called the perimeter of the circle. To find the circumference of any circular thing like a coin, simply wrap any adhesive tape around it such that the end point of tape must meet the starting point. Now unfold the tape and again paste on any flat surface then measure the length of the tape to find the circumference of that circular thing.

We observe some following figures of the circles whose circumference and diameters have been found by the methods given above.








Here, we can also calculate the ratio between circumference and diameter of the circles given above by finding the value $\frac{c}{d}$ where "c" is the circumference and "d" is the diameter of the circles *A, B* and *C* as given in the following table.

| Circles | Circumference (c) | Diameter (d) | Ratio (c/d) |
|---|---|---|---|
| *A* | 7.6 | 2.4 | 3.1666 |
| *B* | 12.57 | 4 | 3.1425 |
| *C* | 19.48 | 6.2 | 3.1419 |

We can see that the ratio between circumference and diameter is approximately the same. We denote this constant value by a Greek symbol $\pi$ the value of which is taken approximately equal to $\frac{22}{7}$ or 3.14.

So, we can write the above statement as, $\frac{\text{circumference } (c)}{\text{diameter } (d)} = \pi$

Or simply we can write it as, $\frac{c}{d} = \pi$

Therefore, $c = d\pi$

But we know that $d = 2r$

So, $c = 2\pi r$

Hence, $c = d\pi$ or $2\pi r$ where '$c$' is the circumference, '$d$' is the diameter and '$r$' is the radius.

## 12.1.2 Finding the Circumference of a Circle Using Formula

**Example 1:** Find the circumference of a circle with diameter 3.2cm.

**Solution:**

Diameter ($d$) = 3.2cm    Circumference (c) = ?

Using the formula, $c = d\pi$

$$c = 3.2 \times \frac{22}{7} = 10.06cm \ (\textit{up to two decimal places})$$

**Example 2:** The radius of a circle is 4.7cm. Find its circumference.

**Solution:**

Radius ($r$) = 4.7cm     Circumference ($c$) = ?

Using the formula, $c = 2\pi r$

$$c = 2\times\frac{22}{7}\times 4.7 = 29.54cm\ (\textit{two decimal places})$$

**Example 3:** The circumference of a circle is 418cm. Calculate the diameter and radius of the circle.

**Solution:**

Circumference ($c$) = 418cm     Radius ($r$) = ?     Diameter ($d$) = ?

(i) Using the formula, $c = 2\pi r$

$$r = \frac{c}{2\pi} = \frac{418\times 7}{2\times 22} = 66.5cm$$

$r$ = 66.5cm

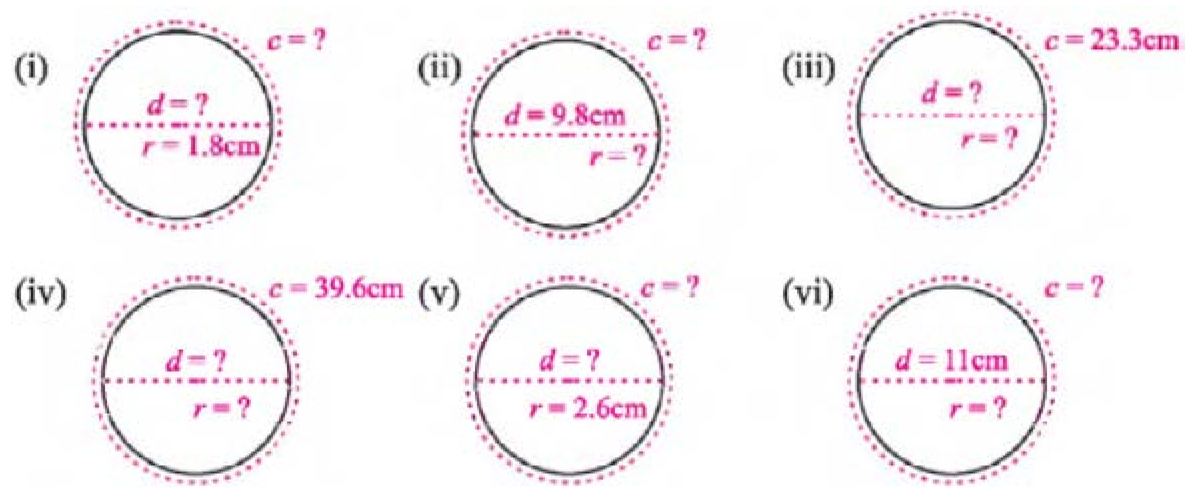
(ii) Using the formula, $c = \pi d$

$$r = \frac{c}{\pi} = \frac{418\times 7}{22} = 133cm$$

$d$ = 133cm

## EXERCISE 12.1

1. Find the unknown quantity when $\pi = \frac{22}{7}$








2. The diameter of a circle is 11.6cm. Find the circumference of the circle.
3. The radius of a circle is 9.8 cm. Find the circumference of the circle.
4. The circumference of a circle is 1.54cm. Find the diameter and radius of the circle (when $\pi = \frac{22}{7}$).
5. The circumference of a circular region is 19.5cm, find its diameter and radius (when $\pi \approx 3.14$).

### 12.1.2 Area of a Circular Region

The area of a circular region is the number of square units inside the circumference of the circle. We know that if we have the measurements of the length and the width of a rectangle, we can find the area of the rectangle by the following formula.

Area of a rectangle = Length x Width

Here we use the same formula to calculate the area of a circle. To make it clear, consider a circle of any suitable radius as shown in the figure (a).

Now we divide the above circular region into 8, 16 and 32 equal parts and rearrange its radial segments after cutting them carefully, as given below.

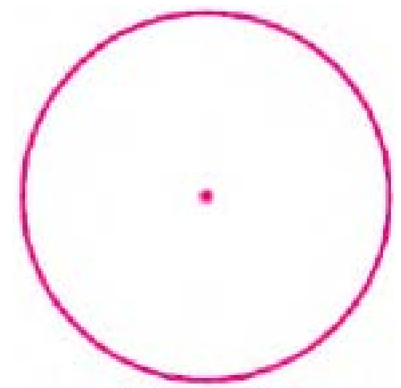
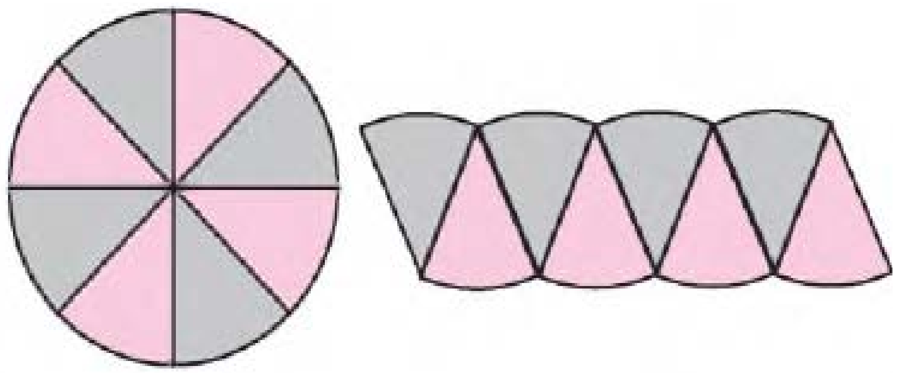


figure (a)

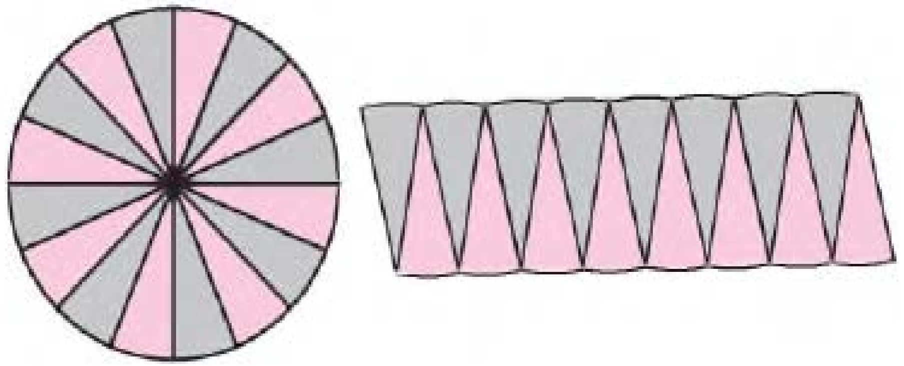
(i) If we divide this circular region into 8 equal parts and rearrange their radial segments, we get the following figure (b).

figure (b)



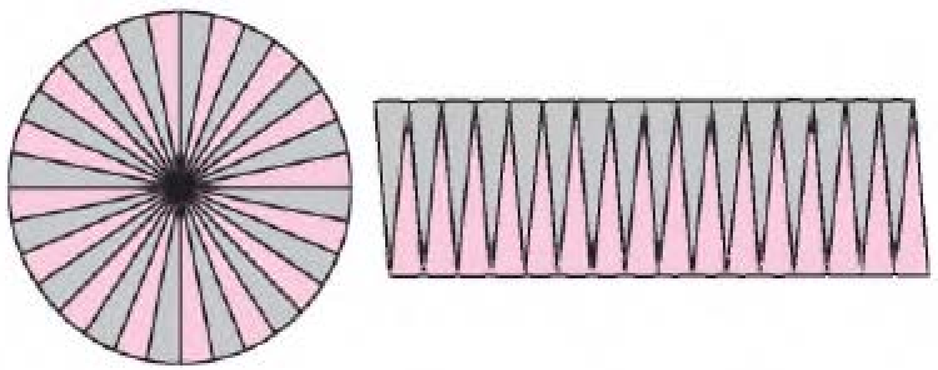
(ii) If we divide this circular region into 16 equal parts and rearrange their radial segments, we get the following figure (c).

figure (c)



(iii) If we divide this circular region into 32 equal parts and rearrange their radial segments, we get the following figure (d).

figure (d)



On considering these figures we see that these figures are of parallelogram in which the edges of half of the sectors are upwards and half of the sectors are downwards and which are adjacent to each other. Similarly, if we continue the division of circular region into more sectors, then this figure will completely be converted into rectangle. Otherwise if we divide the last radial segment into two equal parts and placed them at the both ends of rectangle, then we can get the same rectangle.





In this way we get the length of rectangle which is half of the circumference $\left(\frac{2\pi r}{2}\right)$ of circle and width of the rectangle is equal to the radius of the circle. The length of the rectangle is half of the circumference of the circle because half of the radial segments are upwards and half are downwards and the total length of these all segments is equal to the circumference of the circle.

Length of the rectangle = $\frac{1}{2}$ circumference of the circle

$$=\frac{1}{2}(2\pi r)=\pi r$$

Width of the rectangle = Radius of the circle = r

Area of the circular region = Area of the rectangle

= Length x Width

= $\pi r \times r = \pi r^2$

Therefore,

Area of the circular region = $\pi r^2$

**Example 1:** The radius of a circle is 14.3cm. Find the area of the circle.

**Solution:**

Radius (r) = 14.3cm Area of the circle = ?

Using the formula,

Area of the circle = $\pi r^2$

$$=\left(\frac{22}{7}\times 14.3\times 14.3\right)cm^2$$

= $642.68cm^2$

**Example 2:** The area of a circle is $172.1cm^2$ . Find the circumference of the circle.

**Solution:**

Area of the circle = 172.1cm$^2$ Circumference (c) = ?

We know that circumference = $2\pi r$ and we can calculate the radius of the circle from its area.

Area of the circle = $\pi r^2$

$$172.1 = \frac{22}{7} r^2$$

$$r^2 = \frac{172.1 \times 7}{22} cm^2$$

$$r^2 = 54.76cm^2$$

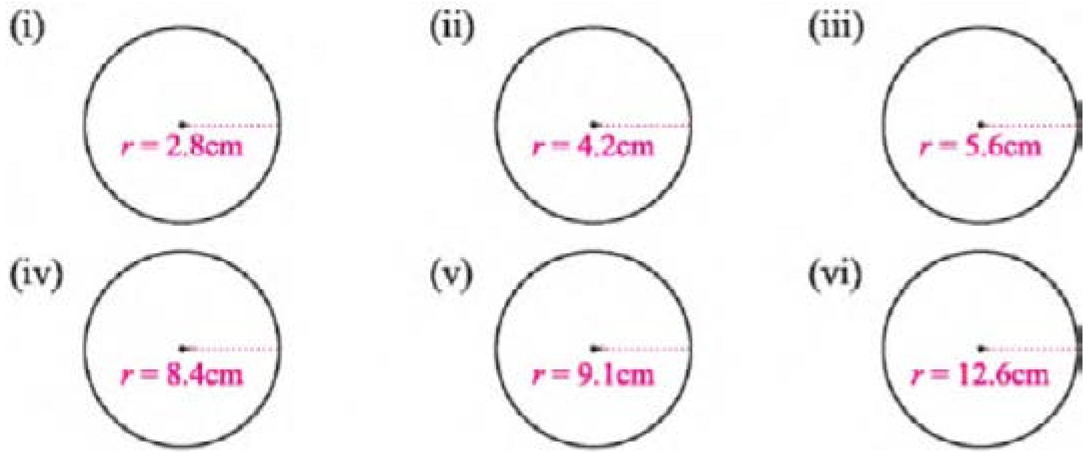
$$r = 7.4 \text{ cm}$$

So, $c = 2\pi r$

$$\text{Circumference } (c) = 2 \times \frac{22}{7} \times 7.4 = 46.51cm$$

## EXERCISE 12.2

1. Find the area of each of the following circles




2. Find the area of a circle whose circumference is 31.43cm.
3. The radius of a circle is 6.3cm. Calculate the area and circumference of the circle.
4. The circumference of a circle is 26.4cm. Find the area of the circle.
5. Find the circumference of a circle whose area is 38.5m$^2$.

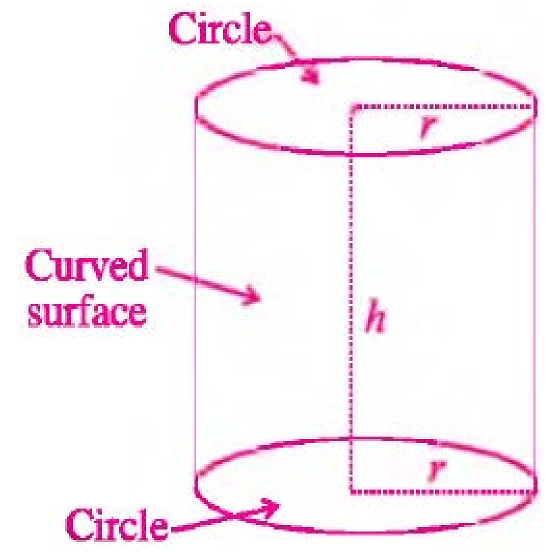




## 12.2 Cylinder

We are already familiar with the shape of a cylinder in our everyday life. Tin pack of soft drinks, pine apple's slice jar, ghee tins, oil drums, chemical drums, different types of rods and pipes, all are examples of a cylinder. For further detail, we examine the following figure (a) of a cylinder.

figure (a)




From the given figure (a), we can observe that a cylinder is a solid which consists of three surfaces of which two are circles of the same radius and one is curved surface. We can also see that two circular region of a cylinder are parallel to each other and the circumference of the circles is the width of the curved surfaces. The length of the curved surface is called the height of the cylinder which can be denoted by "h" where we already know that "r" is the radius of both circles and "d " is the diameter, i.e.

Radius = *r* Diameter = *d* Height = *h*

### 12.2.1 Surface Area of the Cylinder

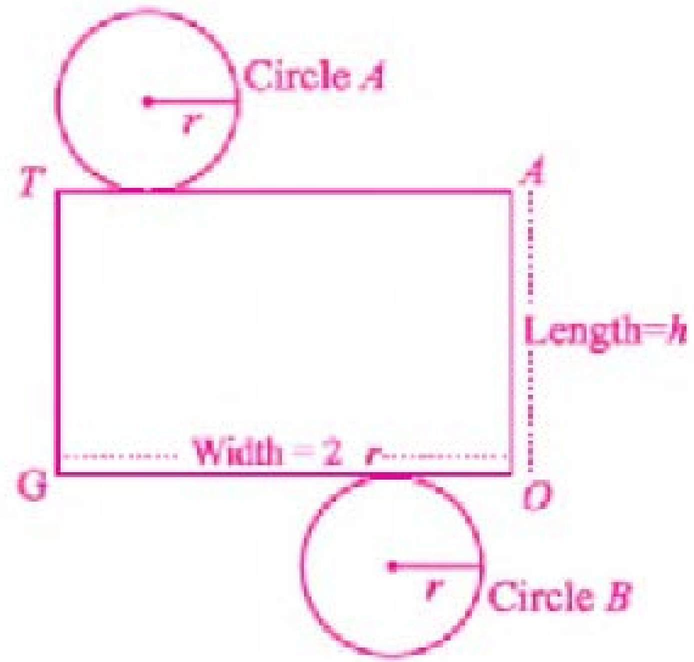
We have already learnt the formula of finding the area of a rectangle and a circle which are given below,

area of a rectangle = length x width

area of a circle = $\pi r^2$

Here we shall use the same formula for finding the surface area of a cylinder. We know that a cylinder is the sum of three flat surfaces *(two circles and one curved surface)* that can be shown by unfolding a cylinder as given in the following figure (b).

figure (b)




From the figure (b), we can examine the three flat surfaces of the cylinder. In which circle *A* is the top and circle *B* is the base of the cylinder where rectangle *GOAT* is the curved surface that if we roll up and join its two edges *GT* and *OA* we get again the same curved surface. Now we can calculate the surface area of a cylinder by finding the sum of areas of two circles *A* & *B* and area of the rectangle *GOAT* as given below:

Width of a rectangle = circumference of a circle = $2\pi r$

Length of a rectangle = $h$

Area of a rectangle *GOAT* = Length x Width

$= h \times 2\pi r = 2\pi rh$

We know that,

Area of a curved surface = area of a rectangle *GOAT*

So, area of a curved surface = $2\pi rh$

Area of a circle *A* = $\pi r^2$

Area of a circle *B* = $\pi r^2$





Area of two circles = area of circle A + area of circle B

$= \pi r^2 + \pi r^2 = 2\pi r^2$

Thus, Surface area of a cylinder = area of a curved surface + area of two circles

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

**Example 1:** Find the surface area of a cylinder with length 18.5cm and radius 3.2cm.

**Solution:**

Length (h) = 18.5cm Radius (r) = 3.2cm

Surface area of a cylinder = ?

Using the formula,

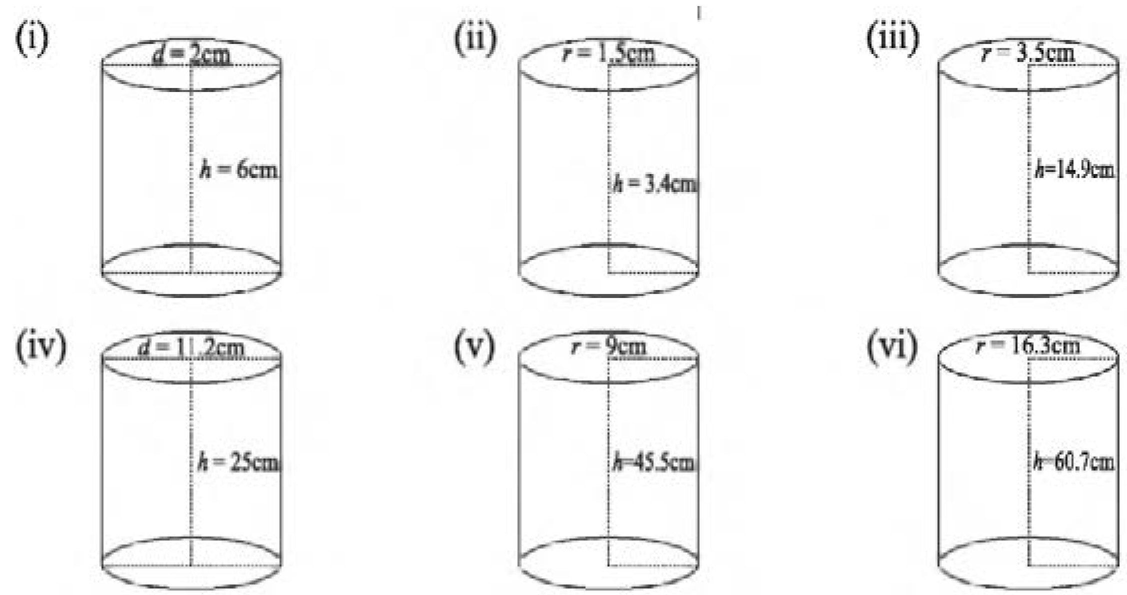
$$\text{Surface area of a cylinder} = 2\pi r(h+r)$$

$$= (2\times\frac{22}{7}\times 3.2(18.5+3.2)cm^2$$

$$\text{Surface area of a cylinder} = (2\times\frac{22}{7}\times 3.2\times 21.7) = 436.48cm^2$$

## EXERCISE 12.3

1. Find the surface area of the following cylinders.



2. The radius of a cylinder is 1.4cm and length is 5.2cm. Calculate the surface area of the cylinder.
3. Find the surface area of a 7.4cm long iron rod of 3.1cm radius.
4. A cylinder is 5m long and radius of the cylinder is 5.3cm. Calculate the surface area of the curved surface.
5. The diameter of a cylinder is 18.5cm and length is 6.1m. Find the surface area of the curved surface.

### 12.2.2 Volume of a Cylinder

We know that to find the volume of any object, we use the measurements of three dimensions and the formula for finding the volume of an object is:

Volume = length x breadth x height

Now by using the above formula of volume, we shall discover another formula for finding the volume of a cylinder. To discover the formula for finding the volume of a cylinder, we can use the following two methods.

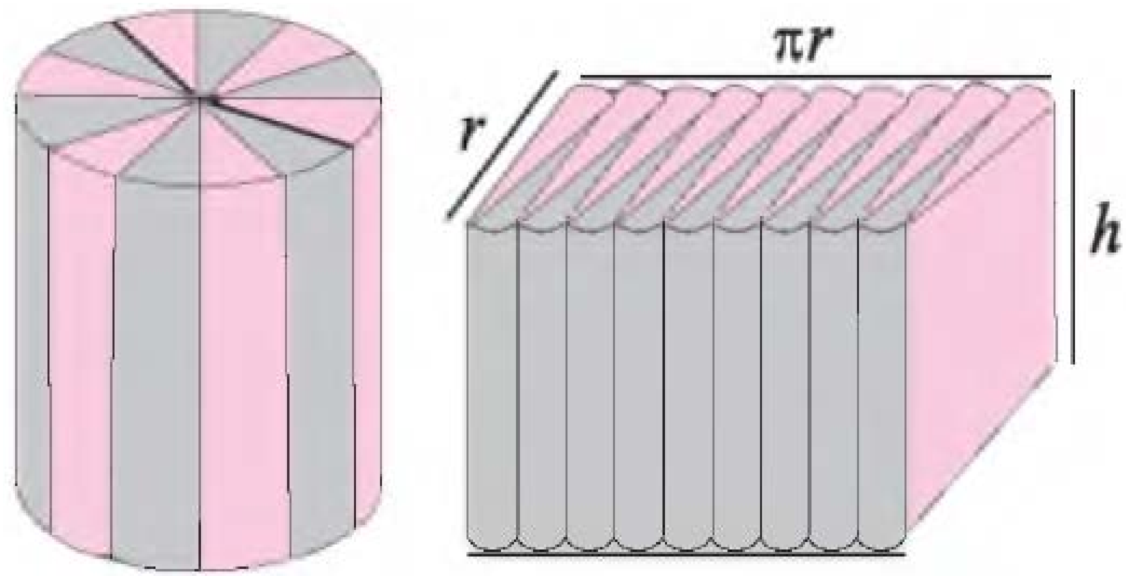
- By making a cuboid
- By stacking coins

- **By making a Cuboid**

We have already learnt under the topic "area of a circle" that when we divide a circle into several but even number of parts and rearrange them according to the instructions we get a rectangle whose length is half of the circumference ($2\pi r$) of the circle and width is equal to the radius (r) of the circle. Then by using the formula for the area of a rectangle, we find the area of a circle. Similarly, we can use the same above method for a cylinder but when we cut a cylinder into several but even number of parts and rearrange them we get a cuboid as shown in following figure.








From the above figure, we can examine that the length of the cuboid is half of the circumference ($2\pi r$) of a circular region, height is equal to the length (h) of the cylinder and breadth is equal to the radius of the circular region. So, if we calculate the volume of the cuboid that will be equal to the volume of the cylinder as given below:

- length of the cuboid = half of the circumference

$$\because \quad \text{length} = \frac{1}{2}(2\pi r) = \pi r$$

- breadth of the cuboid = radius of the circular region

$\therefore$ abreadth = r

- height of the cuboid = length of the cylinder

$\therefore$ aheight = h

- volume of the cylinder = volume of the cuboid

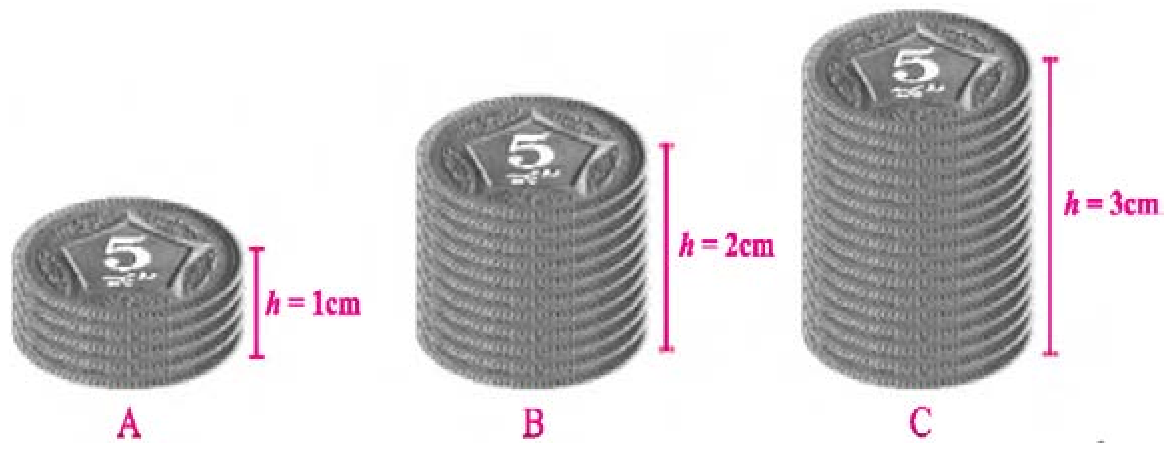
= length x breadth x height

= $\pi r \times r \times h$

$\therefore$ volume = $\pi r^2 h$

- **By stacking coins**

Take 5 coins of Rs.5 and stack up a pile which gives us a shape of a small cylinder *A* of 1cm height as given below. We can check this measurement by stacking up a pile of 5 coins of Rs.5 Similarly, stack up two more piles of 10 coins and 15 coins of Rs.5 that gives us two cylinders of 2cm and 3cm respectively, label them as *B* and *C* as shown in the following figures.




Suppose that the radius of the coin is r then the area of circular region will be $\pi r^2$, i.e.,

Area of the circular region = length x breadth = $\pi r^2$

Where height = 1cm, 2cm and 3cm

Using the above information, we one by one calculate the volume of cylinder *A*, *B* and *C*.

Volume of the cylinder *A* = (length x breadth) x height

= ($\pi r^2$ x 1) $cm^3$ = $\pi r^2$ $cm^3$

Volume of the cylinder *B* = (length x breadth) x height

= ($\pi r^2$ x 2) $cm^3$ = $2\pi r^2$ $cm^3$

Volume of the cylinder *C* = (length x breadth) x height

= ($\pi r^2$ x 3) $cm^3$ = $3\pi r^2$ $cm^3$

We consider that we stack up a pile of some coins whose height is *h* then the volume of cylinder is:

Volume of cylinder = (length x breadth) x height

= ($\pi r^2$ x *h*)

Therefore, volume = $\pi r^2 h$

**Example 1:** Find the volume of a cylinder whose height is 18.5cm and radius is 4.2cm.

**Solution:**

Radius (r) = 4.2cm Height (h) = 18.5cm Volume (v) = ?

Using the formula,

Volume (v) = $\pi r^2$ h

$$=(\frac{22}{7}\times 4.2\times 4.2\times 18.5 cm^3 = 1025.64 cm^3$$





**Example 2:** Find the height of a cylinder whose volume is 3,168cm² and radius is 6cm.

**Solution:**

Radius (*r*) = 6cm Volume (*v*) = 3168$cm^3$ Height(*h*) = ?

Using the formula,

Volume = $\pi r^2 h$

$$h=\frac{\text{volume}}{\pi r^2}=\left(\frac{3168\times 7}{22\times 6\times 6}\right)cm$$

Height = 28 cm

**Example 3:** Find the radius of a cylinder whose height is 14cm and volume is 891 $cm^3$.

**Solution:**

Volume(*v*) = 891$cm^3$ Height(*h*) = 14cm Radius(*r*) = ?

Using the formula,

Volume = $\pi r^2 h$

$$r=\frac{\text{volume}}{\pi h}$$

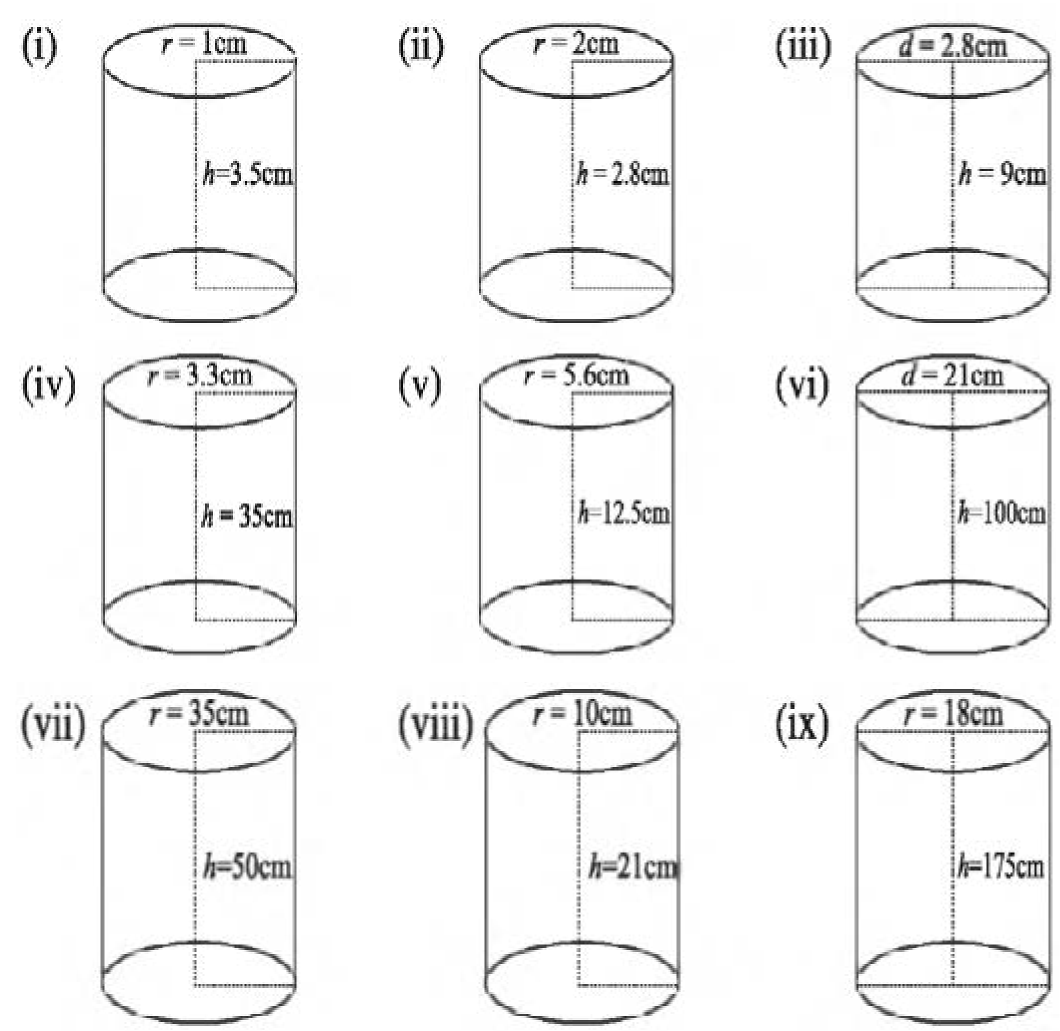
$$=\left(\frac{891\times 7}{22\times 14}\right)cm$$

= 20.25$cm^2$

$$r=\sqrt{20.25}cm=4.5cm$$

## EXERCISE 12.4

1. Find the volume of the following cylinders.




2. Find the volume of a cylinder whose height is 9.8cm and radius of 5.6cm.
3. The volume of a cylinder is 311.85cm$^3$ and height is 10cm. Find the radius of the circular region of the cylinder.
4. The radius of cylinder is 7cm and its volume is 2,233cm$^3$. Find the height of the cylinder.
5. Find the radius of a cylinder when its height is 9.2cm and its volume is 5,667.2cm$^3$

### 12.2.3 Real Life Problems

- **Solving Real Life Problems involving Circumference and Area of a Circle**





**Example 1:** The radius of the wheel of a car is 0.28m. Find in how many revolutions the car will cover a distance of 880 meter.

**Solution:**

Radius($r$) = 0.28m Circumference($c$) = ? Using the formula, $c = 2\pi r$

$$c = 2\times\frac{22}{7}\times 0.28m$$

$c$ = 1.76m

The car will cover the distance of 1.76m in a complete revolution of its wheels. So,1.76m distance covered by car = 1 revolution.

$$\text{880m distance will be covered by a car} = \frac{1}{1.76}\times 880 = 500 \text{ revolutions.}$$

**Example 2:** The diameter of the wheel of Ahmed's bicycle is 0.72m. The bicycle wheel completes 750 revolutions when Ahmed comes from school to house. Find the distance between school and house.

**Solution:**

Diameter (d) = 0.72m Circumference (c) = ?

Using the formula, $c = \pi d$

$$= \frac{22}{7}\times 0.72 = 2.26m$$

In 1 revolution the distance covered = 2.26m

In 750 revolutions the travel is = 2.26 x 750 = 1695m

**Example 3:** The length of the minute hand of a time clock is 3.5cm. Find the distance covered by the pointer of minute hand in 3 hours.

**Solution:**

The length of the minute hand (radius) = 3.5cm Circumference = ?

Using the formula, $c = 2\pi r$

$$= (2\times\frac{22}{7}\times 3.5)\text{cm} = 22cm$$

We know that in 1 hour, pointer of minute hand completes one revolution. So,

In 1 hour, pointer of minute hand covers the distance = 22cm

In 3 hours, pointer of minute hand covers the distance = 3 x 22cm = 66cm

**Example 4:** The circumference of a circular floor is 55m. Calculate the area of the floor and also find the cost of flooring at the rate of Rs.90/$m^2$.

**Solution:**

Circumference (c) = 55m Area of the floor = ?

We know that,

c = $2\pi r$

$$r = \frac{c}{2\pi}$$

$$\text{Radius of circular floor} = \left(\frac{55\times 7}{2\times 22}\right) = 8.75\ metre$$

Now we calculate the area of floor.

Area of the circular region = $\pi r^2$

$$= \left(\frac{22}{7}\times 8.75\times 8.75\right) = 240.62m^2$$

The cost of I$m^2$ = 90 rupees

The cost of 240.62$m^2$ = (90 x 240.62) = Rs. 21655.8

## EXERCISE 12.5

1. The diameter of the wheel of Irfan's bike is 0.7m. The wheel completes 1800 revolutions when he reaches home from the office. Find the distance between Irfan's house and office.
2. The radius of a truck wheel is 0.55m. Calculate how much distance the truck will cover in 1,500 revolutions of the wheel.





3. The length of the minute hand of a watch is 1.75cm. Find in how many hours, the pointer of minute hand will move to cover 165cm.
4. The length of the hour hand of a watch is 1.2cm. Find the distance covered by the hour pointer of hand in 24 hours. *(Hint: The hour hand completes one revolution in 12 hours)*
5. The radius of a circular garden is 24.5m. Find the cost of fencing the garden at the rate of Rs.175 per meter.
6. The diameter of a circular room is 4.2m. Find the cost of flooring at the rate of Rs.150/$m^2$.
7. Find the wages of grass cutting of a circular park at the rate of Rs.5/$m^2$, where the radius of the park is 105m.
8. The radius of a circular pool is 10.5m. Calculate the cost of flooring tiles used on the floor of the pool at the rate of Rs. 180/$m^2$.
9. The diameter of a circular playground is 21m. Calculate the cost of repairing the floor of the playground at the rate of Rs.230/$m^2$ and also find the cost of fencing the playground at the rate of Rs.75/m.

- **Solving Real Life Problems involving Surface Area and Volume of a Cylinder**

**Example 1:** The length of a steel pipe is 2.1m and the radius is 8cm. Calculate its surface area if pipe is open at both the ends.

**Solution:**

The pipe has only curved surface so,

Length (h) = 2.1m = (2.1 x 100) = 210cm

Radius (r) = 8cm

Area of a curved surface = ?

Using the formula,

Area of a curved surface = $2\pi rh$

$$= \left(2\times\frac{22}{7}\times 8\times 210\right) = 10560cm^2$$

**Example 2:** Find the surface area of an oil drum whose length is 1.6m and the diameter is 63cm.

**Solution:**

Length (h) = 1.6m = (1.6 x 100)cm = 160cm

$$\text{Radius } (r) = \frac{diameter(d)}{2} = \frac{63}{2}cm = 31.5cm$$

Surface area of the drum = ?

Using the formula,

Surface area of the drum (cylinder) = $2\pi r\,(h + r)$

$$= 2\times\frac{22}{7}\times 31.5\ (160+31.5)$$

$$= 2\times\frac{22}{7}\times 31.5\times 191.5$$

Surface area of the drum = $37917cm^2$

**Example 3:** Height of an oil drum is 250cm its radius is 70cm. Find the volume or capacity of cylinder in litre.

**Solution:**

Height (h) = 250cm Radius (r) = 70cm

Capacity of an oil drum in litres = volume = ?

Volume of cylinder $= \pi r^2 h$

$$= (\frac{22}{7}\times 70\times 70\times 250)cm^3$$

Volume of cylinder $= 3{,}850{,}000cm^3$

We know that:

$1{,}000cm^3 = 1$ liter

$$\text{Volume (litre)} = \frac{3{,}850{,}000}{1{,}000}\text{litres} = 3{,}850 \text{ litres}$$

## EXERCISE 12.6

1. A cylindrical wooden piece is 19.4cm long. Find the surface area of the wooden piece if its diameter is 14cm.
2. A tin pack of a soft drink is 10cm long and the radius of the tin pack is 3.3cm. Find the surface area of the tin pack.
3. A circular pillar of 22.5cm radius is 6.3m long. Calculate the surface area of the pillar.





4. A cylindrical chemical drum is 220.5cm long and the radius of the drum is 42cm. Calculate the cost of painting the drum at the rate of Rs.0.15/$cm^2$.
5. Radius of a round swimming pool is 17.5m and the depth of the pool is 3m. Calculate the cost of tiles used on the wall of the pool at the rate of Rs. 120/$m^2$.
6. The internal diameter of a round mosque is 31.5m and height of walls is 7m. Find the cost of cementing the round wall of the mosque at the rate of Rs.l9/$m^2$.
7. A cylindrical water tank is 7.7m high and its inner radius is 5m. Calculate the price of marble used in the inner side of the tank at the rate of Rs.500/$m^2$.
8. Find the height of an oil drum whose volume is 12,474m3 and radius is 6.3m.
9. A cylindrical tin can is 77cm high and its radius is 20cm. Find how many liter of oil may be contained in the tin can.
10. Find the capacity of a circular water tank in liters when the height of the tank is 420cm and its diameter is 510cm.

## REVIEW EXERCISE 12

1. Answer the following questions.
   (i) Define the circumference of a circle.
   (ii) What is an area of a circular region?
   (iii) Write the formula for finding the surface area and volume of a cylinder.
   (iv) Write the formula for finding the circumference and area of a circle.
   (v) What is the approximate value of $\pi$?

2. Fill in the blanks.
   (i) The ______ of a circle is the measurement of its closed curve.
   (ii) Two circular regions, of a cylinder are______to each other.
   (iii) The length of the______is called the height of the cylinder.

(iv) The ratio between circumference and diameter of a circle is denoted by the symbol______.

(v) Surface area of a cylinder = area of the curved surface + _______.

3. Tick ( ✓ ) the correct option.

4. Find the area and circumference of the circle, if $\pi = \frac{22}{7}$ and radius is:

(i) 2.8cm (ii) 4.9cm (iii) 10.5cm

(iv) $10\frac{1}{2}cm$ (v) $6\frac{1}{2}cm$

5. Find the surface area and volume of the following cylinders.

(i) r = 14cm, h = 15cm (ii) r = 3.5cm, h = 100cm

(iii) r = 10cm, h = 21cm (iv) r = 4cm, h = 12cm

6. Area of a round bed of roses is 7.065m . Find the cost of fencing around it at the rate of Rs.20 per meter (when $\pi \approx 3.14$).

7. The radius of the wheel of Aslam's cycle is 35cm. To reach school from house, the wheel completes 1200 rounds. Find the distance from house to school (when $\pi \approx \frac{22}{7}$).

8. Find the surface area of 2m 99 long drum whose radius of base is 21cm (when $\pi \approx \frac{22}{7}$).

9. A well is 20m deep and its diameter is 4m. How much soil is required to fill it. (when $\pi \approx 3.14$).





10. Find the cost of spraying a chemical in a circular field at the rate of Rs.10/$m^2$ where the radius of the circular field is 73.5m and also calculate the cost of making hurdle around the field at the rate of Rs.25/m.

## SUMMARY

- The circumference of a circle is the distance around the edge of the circle.
- The ratio between circumference and diameter of a circle is denoted by a Greek symbol $\pi$ whose approximate value is 3.14.
- The area of a circular region is the number of square units inside the circle.
- A cylinder consists of three surfaces i.e. two circles of same radius and one curved surface.
- C = d$\pi$ or 2$\pi$r, where 'c' is the circumference, 'd' is the diameter and 'r' is the radius.
- Area of the circular region = $\pi r^2$
- Surface area of a cylinder = $2\pi r (h + r)$
- Volume of a cylinder = $\pi r^2 h$

*Version: 1.1*

CHAPTER

# 13 INFORMATION HANDLING

*Animation 13.1: Information Handling*
*Source & Credit: eLearn.Punjab*

### Student Learning Outcomes

**After studying this unit, students will be able to:**

- Demonstrate data presentation.
- Define frequency distribution (i.e. frequency, lower class limit, upper class limit, class interval).
- Interpret and draw pie graph.

## Introduction

In the world around us, there are a lot of questions and situations that we want to understand, describe, explore and access. For example,

- How many hospitals are there in different cities of Pakistan?
- How many children were born during the last 10 years?
- How many doctors will be required in the next 5 years?

To know about such things, we collect information and present it in a manageable way so that useful conclusions can be drawn. The branch of statistics that deals with this process is called information handling.

## 13.1 Data

Data means facts or groups of information that are normally the results of measurements, observations and experiments. These results help us in reviewing our past performance and future planning. For example, the government of a state prepares its budgets and development plans on the basis of a collected data about the resources and population.

### 13.1.1 Presentation of Data

After the collection of a data, the most important step is its presentation that provides basis to draw conclusions. Data can be





represented in the form of tables and different kinds of graphs.
We know that a data is collected in raw form and it provides us information about individuals. Data in such form is called ungrouped data. After arranging the data for desired information, it is called grouped data. For example, a teacher collected the score of 20 students in mathematics test:
11, 52 ,40 ,95 , 65 ,45 , 35 , 30 , 88, 56, 75, 90, 81, 82, 28, 49, 67, 98, 64, 92
This is an ungrouped data. Now if we arrange it to represent information into groups, then it is called grouped data.

- Number of students who scored from 11 to 40 = 5
- Number of students who scored from 41 to 70 = 7
- Number of students who scored from 71 to 100 = 8

It can be seen that it is easier to visualize the given information if data is presented in grouped form. We can also represent a grouped data using a table.

| Group | Score | Tally Marks | No. of Students |
|---|---|---|---|
| 11 – 40 | 11, 40, 35, 30, 28 | 𝍸 | 5 |
| 41 – 70 | 52, 65, 56, 45, 49, 67, 64 | 𝍸 \|\| | 7 |
| 71 – 100 | 98, 88, 75, 90, 81, 82, 98, 92 | 𝍸 \|\|\| | 8 |

The method that we used to record the results in the table is called tallying in which we draw tally marks according to the number of individuals of a group. We make the set of fives by crossing the four marks with the fifth mark. This makes easy to count the tally marks. For example, to show 12 individuals of a group we draw tally marks 𝍸 𝍸 ||.
We can also characterize the information presented in the example as;

| Scores | Characteristics |
|---|---|
| 71 – 100 | Excellent |
| 41 – 70 | Good |
| 11 – 40 | Poor |

### 13.1.2 Frequency Distribution

The conversion of ungrouped data into grouped data so that the frequencies of different groups can be visualized is called frequency distribution

**The table which shows the frequency of class intervals is called the frequency table.**

- **Frequency**

The number of values that occurs in a group of a data is called its frequency, e.g. in the above given example,

The frequency of (11 – 40) is 5.
The frequency of (41 – 70) is 7.
The frequency of (71 – 100) is 8.

- **Class Limits**

**Upper Class Limit:** The greatest value of a class interval is called the upper class limit, e.g. in the class interval (41 – 70), 70 is the upper class limit.

**Lower Class Limit:** The smallest value of a class interval is called the lower class limit, e.g. in the class interval (71 – 100), 71 is the lower class limit

- **Class Intervals**

Each group of a data is also known as the class interval. For example, (11 – 40), (41 – 70) and (71 – 100) are class intervals. Each interval represents all the values of a group.

**Size of the Class Interval:** The number of values in a class interval is called its size or length. For example, the size or length of class interval (11 – 40) is 30 that can be checked by counting. It can also be calculated by subtracting the lowest value of the data from greatest value and divide the result by the number of class intervals as shown below:

Lowest value = 11
Greatest value = 100

Now use the formula to calculate the size.





$$\text{Size of interval} = \frac{\text{greatest value} - \text{lowest value}}{\text{no. of intervals}}$$

$$= \frac{100-11}{3} = \frac{89}{3} = 29.6$$

Round off the answer, this is the required size of the interval, i.e. $29.6 \simeq 30$

**Example 1:** There are 40 students in the class VII who got the following marks in an English test. Make a frequency table by using 5 classes of an equal size.

35, 9, 26, 41, 27, 15, 18, 60, 46, 33, 24, 15, 52, 39, 28, 89, 74, 68, 56, 38, 92, 49, 28, 82,19 ,21, 34 ,23 ,43 ,77 ,65 ,64 ,21 ,59 ,15, 33, 66, 29, 33,65,

**Solution:**

We know that,

$$\text{Size of class} = \frac{\text{greatest value} - \text{lowest value}}{\text{no. of intervals}}$$

We can see from the above un-grouped data that of:

Greatest value = 92
Lowest value = 9
No. of classes = 5

$$\text{Size of class} = \frac{92-9}{5} = 16.6 \simeq 17 \ (round\ up)$$

| Class Interval | Tally Marks | Frequency |
|---|---|---|
| 9 – 25 | 𝍸 𝍸 | 10 |
| 26 – 42 | 𝍸 𝍸 ||| | 13 |
| 43 – 59 | 𝍸 | | 6 |
| 60 – 76 | 𝍸 || | 7 |
| 77 – 93 | 𝍸 | 4 |

**EXERCISE 13.1**

1. The telephone bills paid by 12 consumers are given below.

Make a frequency table of 5 classes of an equal size.
510, 700, 356, 603,422, 674,481, 545, 718, 592, 685, 569

2. In a board examination, 20 students of the Dawn Public School got the following marks out of 850 marks. Construct a frequency table by taking 100 as a class interval.
551, 786, 678, 725, 788, 580, 720, 690, 750, 651, 599, 609, 719, 760, 625, 775, 646,667,753,675
3. The daily wages of 15 workers are given below. Make a frequency table of 4 classes of an equal size.
400, 225, 250, 380, 425, 175, 230, 325, 150, 300, 200, 180, 350, 375, 200
4. A cricket player made the list of his last 18 innings scores which is given below.
122, 102, 72, 99, 89, 106, 99, 85, 92, 108, 102, 98, 95, 76, 80, 65, 101, 96, Make a frequency table of 6 classes of an equal size.
5. The following data shows the distance in km that was travelled by Mr. Usman in last 21 days.
77, 58, 62, 85, 32, 71, 59, 60, 38, 32, 69, 80, 76, 92, 61, 82, 74, 70, 99, 44, 53 Make a frequency table of 5 classes of an equal size.
6. The following data is showing the sale of a bike company during last months.
571, 692, 700, 533, 832, 744, 649, 584, 613, 735, 872, 900, 512, 864, 654, 782, 777, 555, 632, 880, 628, 529, 680, 756, 567, 548, 824, 719, 678, 721
Make a frequency table by taking 100 as a class interval.

## 13.2 Pie Graph

"The representation of a numerical data in the form of disjoint sectors of a circle is called a pie graph."

A pie graph is generally used for the comparison of some numerical facts classified in different classes. In this graph, the central angle measures 360° which is subdivided into the ratio of the sizes of the groups to be shown through this graph. Following examples will help to understand the concept of a pie graph.





**Example 1:** It is compulsory for each student to take part in the different games. Out of 1800 students in the school 750 play cricket, 200 play badminton, 400 play hockey and 450 play football. In order to represent their comparison, draw a pie graph.

**Solution:**

Total number of students = 1800

(i) Find the angle for each sector by using the following formula

$$\text{Required angle} = \frac{\text{No. of students play a game}}{\text{total students}} \times 360^0$$

$$\text{Measure of angle associated with badminton} = \frac{200}{1800} \times 360^0 = 40^0$$

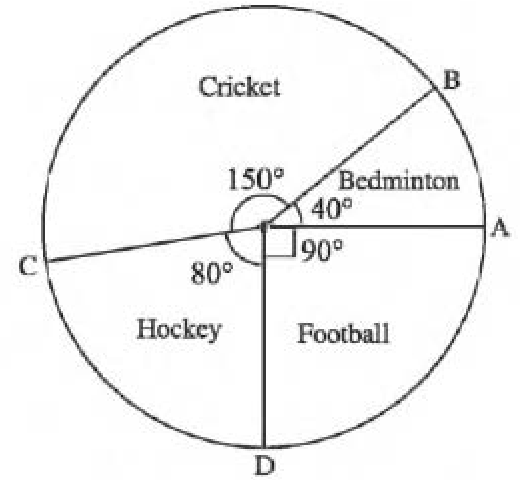
$$\text{Measure of angle associated with cricket} = \frac{750}{1800} \times 360^0 = 150^0$$

$$\text{Measure of angle associated with hockey} = \frac{400}{1800} \times 360^0 = 80^0$$

$$\text{Measure of angle associated with football} = \frac{450}{1800} \times 360^0 = 90^0$$

(ii) In order to draw a pie graph
- Draw a circle of a suitable radius.
- Draw an angle of 40° representing the badminton.
- Draw an angle 150° representing the cricket.
- Draw an angle 80° representing the hockey.
- Remaining angle will be of 90° representing the football.

(iii) Label each sector according to the following figure.




**Have you noticed that students like cricket most?**

**Example 2:** The following table shows the favourite food of the students of the grade VII. Plot a pie graph to show the favourite food of the students.

| Food | Fried Chicken | Mutton Karahi | Biryani | Minced Meat | Vegetables |
|---|---|---|---|---|---|
| No. of Students | 40 | 20 | 10 | 6 | 4 |

**Solution:**

(i) Find the angles for each sector by using the following formula.

(a) Required angle = $\dfrac{\text{No. of students like food}}{\text{total students}} \times 360^0$

(b) Angle for fried chicken = $\dfrac{40}{80} \times 360^0 = 180^0$

(c) Angle for mutton karahi = $\dfrac{20}{80} \times 360^0 = 90^0$

(d) Angle for biryani = $\dfrac{10}{80} \times 360^0 = 45^0$

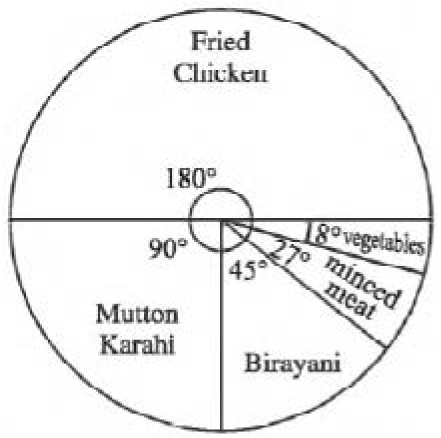
(e) Angle for minced meat = $\dfrac{6}{80} \times 360^0 = 27^0$

(f) Angle for vegetables = $\dfrac{4}{80} \times 360^0 = 18^0$

(ii) Draw a circle of any suitable radius.

(iii) Divide the circle into the sectors of calculated angles.

(iv) Label each sector according to the following figure.




**Did you see that fried chicken is the most favourite food of the students?**





## EXERCISE 13.2

1. Hina went for shopping and spent 30% of her pocket money for food, 35% on buying books, 20% on school dress and saved 15%. Represent the data on pie graph.

2. A media reporter conducted a survey of persons visiting market during the two hours. He found that there were 720 persons visited the market out of which 320 were women, 220 men and 180 children. Draw a pie graph.

3. In a class, the grades obtained by the students in the final examination are given below. Draw the pie graph.

| Grade | A+ | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| No. of students | 2 | 6 | 10 | 30 | 6 | 4 | 2 |

4. Details of students in five classes of a school are given below. Draw a pie graph to show the comparison.

| Class | I | II | III | IV | V |
|---|---|---|---|---|---|
| No. of students | 300 | 270 | 225 | 150 | 135 |

5. Noreen has the following types of books in her library. Draw pie graph showing the information.

| Subject | English | Islamic | Stories | Poems |
|---|---|---|---|---|
| No. of books | 180 | 90 | 60 | 30 |

## REVIEW EXERCISE 13

1. Answer the following questions.
   (i) What is meant by the grouped data?
   (ii) Define a class interval.
   (iii) Define a pie graph.
   (iv) Write the formula for finding the size of class interval.
   (v) Which method is called tallying

2. Fill in the blanks.
   (i) _______ means groups of information that are normally the results of measurements, observations and experiments.
   (ii) Each________represent all the values of a group.
   (iii) A data is collected in _______ form and it provides us information about individuals.
   (iv) The method which is used to record the result is called______.
   (v) The greatest value of a class interval is called the ___ limit.
   (vi) The number of values in a class interval is called its _______.
   (vii) The representation of a numerical data in the form of disjoint sectors of a circle is called a _______.

3. Tick ( ✓ ) the correct option.

4. The ages of patients in years admitted in a hospital during a week are given below. Group the data taking 10 as the size of an interval.
   25, 50, 49, 47, 26, 10, 2, 1, 15, 17, 18, 19, 27, 28, 30, 35, 40, 37, 32, 31, 3, 4, 7, 10, 15, 12, 13, 17, 14, 20, 22, 24, 26, 30, 17, 35, 40, 36, 32, 31, 37

5. The following data shows the distance in km that Mr. Ghani traveled in last month.
   90, 44, 15, 19, 28, 9, 92, 17, 8, 84, 50, 60, 77, 69, 24, 89, 63, 74, 35, 48, 39, 81, 58, 37, 55, 67, 46, 30, 26, 79.
   Construct the frequency table of 6 classes of an equal size.

6. Ali and his friends eat breads in a day as shown in the table.

| Meals | Breakfast | Lunch | Dinner | Supper |
|---|---|---|---|---|
| No. of Breads | 12 | 24 | 16 | 8 |





By using the table, draw a pie graph.

7. In a party, a host served the guest by following food items.

| Food Items | Cold Drink | Sandwich | Burger | Cake |
|---|---|---|---|---|
| Quantity | 180 | 124 | 330 | 86 |

Use the table to draw a pie graph.

## SUMMARY

- Data means facts or groups of information that are normally the results of measurements, observations and experiments.
- Data is collected in raw form and it provides us information about individuals such form of the data is called ungrouped data.
- In a grouped data, each group is also known as the class interval.
- The greatest value of a class interval is called the upper class limit.
- The smallest value of a class interval is called the lower class limit.
- The number of values that occurs in a class interval is called its frequency.
- The table which shows the frequency of class intervals is called frequency tables.
- The representation of a numerical data in the form of disjoint sectors of a circle is called a pie graph.
- In pie graph, the central angle measures 360° which is subdivided into the ratio of the sizes of the groups.